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THE MEASUREMENT OF THE THERMAL DIFFUSIVITY OF SOLIDS

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ABSTRACT

This paper presents an experimental method for the determination of the thermal diffusivity of solids. The analytical expression is match with the experimental results through the value of the thermal diffusivity of the solid. Easy to implement, this method can be used as a laboratory of a heat transfer course.

Keywords: thermal diffusivity, experiment, thermal property

1. INTRODUCTION

The measurement of the physical properties of a solid is a very good opportunity for the students to understand in a better way the physical phenomena. In this category, the thermal diffusivity measurement is a classical example used in the heat transfer laboratories.

This paper presents an example of the way in which the thermal phenomena, and particularly, the thermal diffusivity is studied using a simple experimental set-up.

2. MATHEMATICAL MODEL

Figure 1 presents the experimental set-up used to measure the thermal diffusivity of a solid material. A cylindrical rod is considered as being infinitely long because its length, L, is much bigger than its radius. The rod is heated by a flux of air of constant temperature, T_1, at the abscissa $x = L$ and it is isolated at all the other boundaries. The ambient temperature has a constant temperature, T_0, while h is the convection heat transfer coefficient between the rod and the environment.

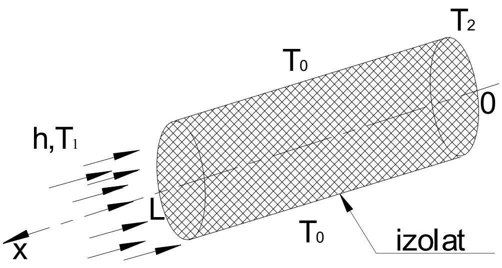

Fig. 1. *The experimental set-up*

The heat transfer equation for the rod:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \tag{1}$$

the initial condition:

$$t = 0,\ T = T_0, \tag{2}$$

along with the boundary conditions:

$$x = 0,\ \frac{\partial T}{\partial x} = 0; \tag{3}$$

$$x = L,\ -k\frac{\partial T}{\partial x} = h(T - T_1), \tag{4}$$

are transformed using the definition of θ:

$$\theta = T - T_1. \tag{5}$$

The new form of the governing equation:

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \tag{6}$$

and the new form of the initial and boundary conditions, Eqs. (7-9):

$$t = 0,\ \theta = T_0 - T_1 = \theta_0 \tag{7}$$

$$x = 0,\ \frac{\partial \theta}{\partial x} = 0; \tag{8}$$

$$x = L,\ -k\frac{\partial \theta}{\partial x} = h\theta \leftrightarrow \quad k\frac{\partial \theta}{\partial x} + h\theta = 0 \tag{9}$$

are solved using the three steps that are well known in the scientific literature [1]:

Step 1: *The direction of the homogenous boundary conditions.*
The boundary conditions, Eq. (8) and Eq. (9), reveal the abscissa "x" as a direction of the homogenous boundary conditions.

Step 2: *The separation of the variables.*

The two variables, the space (x) and the time (t), require us to consider that the dimensionless temperature, θ, is the product of two functions: X(x), a function of space, and τ(t), a function of time:

$$\theta(x,y) = X(x)\cdot\tau(t) \tag{10}$$

Replacing Eq. (10) in Eq. (6),

$$X''\tau = \frac{1}{\alpha}X\tau' \quad , \tag{11}$$

and dividing by Xτ on both sides of Eq. (11), we obtain Eq. (12):

$$\frac{X''}{X} = \frac{1}{\alpha}\frac{\tau'}{\tau} \quad . \tag{12}$$

The left side of Eq. (12) is a function of space, x, while the right side of Eq. (12) is a function of time, t. This situation requires that both sides of Eq. (12) to be constant:

$$\frac{X''}{X} = -\lambda^2 \quad ; \qquad \frac{1}{\alpha}\frac{\tau'}{\tau} = -\lambda^2 \, . \tag{13}$$

The minus sign in Eq. (13) is imposed by the fact that the abscissa x is a direction of homogeneous boundary conditions.

The general solution of Eq. (13) is:

$$\begin{cases} X = C_1\,sin(\lambda x) + C_2\,cos(\lambda x); \\ \tau = C_3 e^{-\lambda^2\alpha t} \end{cases} \tag{14}$$

, where C_1, C_2, C_3 and λ are unknown constants.

Consequently, θ has the following form:

$$\theta = C_3 e^{-\lambda^2\alpha t}\left[C_1\,sin(\lambda x) + C_2\,cos(\lambda x)\right] . \tag{15}$$

The unknowns: C_1, C_2, C_3 and λ are found using the initial and the boundary conditions, Eqs. (7-9).

Equation (8) imposes:

$$\frac{d}{dx}\left[C_1\,sin(\lambda x) + C_2\,cos(\lambda x)\right] = \left.\lambda C_1\,cos(\lambda x) - \lambda C_2\,sin(\lambda x) = 0\right|_{x=0} \tag{16}$$

or

$$C_1 = 0 \tag{17}$$

Equation (18) shows the new form of the dimensionless temperature, θ:

$$\theta = C_2 C_3 e^{-\lambda^2\alpha t}\,cos(\lambda x) \tag{18}$$

Defining $C_2 C_3 = M$, we have the following result:

$$\theta = M e^{-\lambda^2\alpha t}\,cos(\lambda x) . \tag{19}$$

Equation (9) and Eq. (19) lead us to:

$k\,M\lambda e^{-\lambda^2\alpha t}\,sin(\lambda x) = \left.hMe^{-\lambda^2\alpha t}\,cos(\lambda x)\right|_{x=L}$ or $(\lambda L)tg(\lambda L) = \frac{hL}{k}$. Making the notation $a = \lambda L$, we obtain:

$$a\,tg(a) = Bi \tag{20}$$

Here, $Bi = \frac{hL}{k}$, the Biot number, represents the dimensionless heat transfer coefficient between the rod and the environment.

Equation (20) has an infinity of solutions, a_n. θ becomes:

$$\theta = \sum_{n=1}^{\infty} M_n e^{-\lambda_n^2\alpha t}\,cos(\lambda_n x) \tag{21}$$

Step 3: *The orthogonality condition.*
The coefficients M_n are found using Eq. (7) and Eq. (21):

$$\theta_0 = \left.\sum_{n=1}^{\infty} M_n e^{-\lambda_n^2\alpha t}\,cos(\lambda_n x)\right|_{t=0} \tag{22}$$

or

$$\theta_0 = \sum_{n=1}^{\infty} M_n\,cos(\lambda_n x) \tag{23}$$

Multiplying Eq. (23) with $cos(\lambda_m x)$ and integrating between $x = 0$ and $x = L$, we obtain Eq. (24):

$$\int_0^L \theta_0\,cos(\lambda_m x)dx = = \sum_{n=1}^{\infty} M_n \int_0^L cos(\lambda_n x)cos(\lambda_m x)dx \tag{24}$$

Using the orthogonality condition of cosines function:

$$\int_0^H cos\left(n\frac{\pi y}{H}\right)cos\left(m\frac{\pi y}{H}\right)dy = 0 \;if\; m \neq n \, , \tag{25}$$

Eq. (24) becomes:

$$\int_0^L \theta_0 \cos(\lambda_m x)dx = M_m \int_0^L \cos^2(\lambda_m x)dx \,. \tag{26}$$

After solving the integrals of Eq. (26), M_n becomes:

$$M_n = 4\theta_0 \frac{\sin(a_n)}{2a_n + \sin(2a_n)} \tag{27}$$

, while θ, Eq. (21), becomes:

$$\theta = \sum_{n=1}^{N} 4\theta_b \frac{\sin(a_n)}{2a_n + \sin(2a_n)} e^{-\lambda_n^2 \alpha t} \cos(\lambda_n x) \,. \tag{28}$$

The temperature we measure, at the abscissa $x = 0$, T_2, is:

$$T_2 = T_1 + \sum_{n=1}^{N} 4\theta_b \frac{\sin(a_n)}{2a_n + \sin(2a_n)} e^{-\lambda_n^2 \alpha t} \,. \tag{29}$$

3. RESULTS AND DISCUSSIONS

For the experimental set-up presented by Table 1, the first five solutions of Eq. (20) were established and they are presented by Table 2.

Table 1. The experimental set-up

Variable	T_0	T_1	L
Value	24.0°C	190°C	34cm

Table 2. The first five solutions of Eq. (20).

a_1	a_2	a_3	a_4	a_5
0.435	3.205	6.315	9.445	12.585

The measured and the calculated values of the temperature at the isolated end of the rod are presented by Table 3 and Fig. 2 for a Biot number of 0.2 and the thermal diffusivity α=1.08 cm^2/s. This is the Biot number and the thermal diffusivity that assure the best concordance between the measured and the calculated values of the temperature T_2. In this case the standard deviation $SD = \sqrt{\frac{\sum (T_{measured} - T_2)^2}{N-1}}$ (where N is the number of the measurements) has a value of 1.089.

4. CONCLUSIONS

This paper presents an experimental method designed to measure the thermal diffusivity of solids. The experimental set-up and the mathematical modeling are appropriate for a heat transfer laboratory where the students can obtain a better understanding of the thermal phenomena that occur and of the techniques used in the analysis of the non-stationary heat transfer regimes.

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Table 3. The experimental and the numerical data.

Nr. Crt.	t [min]	$T_{measured}$ [°C]	T_2 Eq. (29)
1	2	24.65	23.25
2	4	25.34	26.50
3	6	27.62	29.54
4	8	30.49	32.78
5	10	33.75	36.04
6	12	37.31	39.25
7	14	40.75	42.41
8	16	44.19	45.51
9	18	47.52	48.54
10	22	53.98	54.42
11	24	57.2	57.26
12	26	60.31	60.05
13	28	63.04	62.78
14	30	65.75	65.45
15	34	71.17	70.62
16	36	73.59	73.313
17	38	75.80	75.58
18	40	78.02	77.98
19	42	80.14	80.33
20	50	88.96	89.25
21	52	91.06	91.37
22	54	92.97	93.44
23	56	95.36	95.47
24	58	97.26	97.45
25	60	99.07	99.39

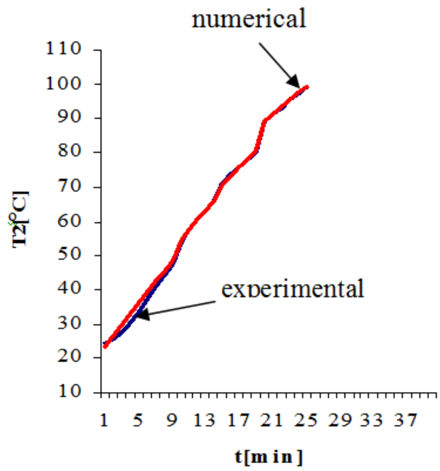

Fig. 2. *The experimental and the numerical data*

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