# OPTIMAL CONTROL DEVELOPMENT SYSTEM FOR ELECTRICAL DRIVES

#### Marian GAICEANU, Emil ROSU, Romeo PADURARU, Cristinel DACHE

Dunarea de Jos University of Galati, Domneasca 47, 800008 Galati, Romania, Marian.Gaiceanu@ieee.org

Abstract: In this paper the optimal electrical drive development system is presented. It consists of both electrical drive types: DC and AC. In order to implement the optimal control for AC drive system an Altivar 71 inverter, a Frato magnetic particle brake (as load), three-phase induction machine, and dSpace 1104 controller have been used. The on-line solution of the matrix Riccati differential equation (MRDE) is computed by dSpace 1104 controller, based on the corresponding feedback signals, generating the optimal speed reference for the AC drive system. The optimal speed reference is tracked by Altivar 71 inverter, conducting to energy reduction in AC drive. The classical control (consisting of rotor field oriented control with PI controllers) and the optimal one have been implemented by designing an adequate ControlDesk interface. The three-phase induction machine (IM) is controlled at constant flux. Therefore, the linear dynamic mathematical model of the IM has been obtained. The optimal control law provides transient regimes with minimal energy consumption. The obtained solution by integration of the MRDE is orientated towards the numerical implementation-by using a zero order hold. The development system is very useful for researchers, doctoral students or experts training in electrical drive. The experimental results are shown.

Keywords: optimal control, matrix Riccati differential equation, electrical drive, induction motor.

#### 1. INTRODUCTION

In many countries, the electrical motors are used in various places: industry and domestic applications. Taking into account that the energy efficiency is an European Union priority, the authors have been developed an experimental system for electrical drive optimization.

The dedicated optimal control developed system includes both drive systems: AC and DC.

A large proportion of electrical energy is consumed by induction motors. Therefore, the electrical energy reduction by just few percent has a major impact in total electrical energy consumption.

The optimal control for three-phase IM drive system have attained researchers' attention for many years (Veerachary, 2002; Tamimi, *et al.*, 2006; Cheng, *et al.*, 2007; Su, *et al.*, 2004; Matinfar, *et al.*, 2005; Deneika, 2006; Zhongke, 2004; Jianqiang, *et al.*, 2007). This paper focuses on the AC drive optimization.

In order to improve the AC drive efficiency, in which dynamic regimes are often required, an optimal control law is computed by using linear quadratic criteria. The objectives of the optimal control law are smooth response, no oscillations on the control interval, no overshoot, the fast compensation of the load torque, and of minimizing the AC drive input energy.

#### 2. PROBLEM FORMULATION

#### 2.1. The model of the three-phase induction motor

The decoupling of the stator current components,  $i_{sd}$  and  $i_{sq}$ , is performed in rotor field reference frame (Athans, *et al.*, 1966; Leonhard, 1996). Maintaining the flux component of the stator current at the constant value, the mathematical model of the IM becomes linear and has the form (Gaiceanu, *et al.*, 1999; Gaiceanu, 2004a; Rosu, *et al.*, 1998a):

$$\begin{pmatrix} 1 \\ \dot{a}_m(t) \\ \dot{q}(t) \end{pmatrix} = \begin{bmatrix} -\frac{F}{J} & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_m(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \frac{K_m}{J} \\ K_q \end{bmatrix} \cdot i_{sq}(t) + \begin{bmatrix} -\frac{1}{J} \\ 0 \end{bmatrix} \cdot m_l(t)$$

with specific constants:

(2) 
$$K_q = \frac{1}{\tau_R i_{mR}}, K_m = \frac{3}{2} p \frac{M}{1 + \sigma_R} i_{mR}$$

in which  $i_{sd}$  is the flux component current;  $i_{sq}$ -the torque component current;  $i_{mR}$  the rotor magnetizing current;  $\omega_m$  the instantaneous mechanical angular velocity of the rotor;  $m_e$  the electromagnetic torque of the induction motor;  $m_1$  load torque; q the angular position of the rotor field; J the combined inertia of the motor and load; F the viscous friction coefficient; M the mutual inductance between the stator and the rotor d, q equivalent windings;  $\tau_R$  the rotor time constant;  $\sigma_R$  the rotor leakage factor; p the number of pole pairs. Eq.1, in the state form, becomes

(3) 
$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{w}(t)$$

where  $\mathbf{x}(t)$  is the state vector  $\mathbf{x}(t) = \begin{bmatrix} \boldsymbol{\omega}_m(t) \\ q(t) \end{bmatrix}$ ,  $\mathbf{u}$  is

the control vector  $\mathbf{u}(t) = [i_{sq}(t)]$  and the perturbation vector is  $\mathbf{w}(t) = [m_t(t)]$ .

2.2. The performance functional quadratic criteria

The performance functional quadratic criteria (Athans, *et al.*, 1966) is as follows:

(4) 
$$J = \frac{1}{2} [\mathbf{x}(t_1) - \mathbf{x}_1]^T \mathbf{S} [\mathbf{x}(t_1) - \mathbf{x}_1]$$
$$+ \frac{1}{2} \int_0^{t_1} [\mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t)] dt$$

in which the required final state is:

(5) 
$$\mathbf{x}_1 = \begin{bmatrix} \boldsymbol{\omega}_m^* \\ 0 \end{bmatrix}$$

and the weighting matrices

(6) 
$$S \ge 0$$
,  $R > 0$ ,  $Q \ge 0$ 

have in view the minimizing of the square error between the reached state and the desired state  $\mathbf{x}_1$  in the fixed time  $t_1$ , the control effort and the expended energy in the motor windings. Therefore, the optimal control problem is: with free-end point, fixed time and unconstrained. The restrictions of the magnitude for the control and state could be solved by the adequate choice of the weighting matrices.

#### 3. THE SOLUTION OF THE OPTIMAL CONTROL PROBLEM

By using the variational method, the Hamiltonian of the optimal control problem is

(7) 
$$H(\lambda, x, u, t) = \frac{1}{2} \left[ x^{T}(t) Q x(t) + u^{T}(t) R u(t) + \lambda^{T}(t) \cdot x(t) \right]$$

 $\lambda(t) \in \Re^2$  being the costate vector.

Therefore, the following canonic system is obtained

(8) 
$$\begin{bmatrix} \overset{\circ}{\mathbf{x}}(t) \\ \overset{\circ}{\boldsymbol{\lambda}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}} \\ -\mathbf{Q} & -\mathbf{A}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \boldsymbol{\lambda}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix} \mathbf{w}(t)$$

The well known boundary conditions result from the initial state  $x(0)=x_0$  and from the transversality of the costate vector (Athans, *et al.*, 1966; Rosu, *et al.*, 1998b): .

(9) 
$$\lambda(t_1) = \mathbf{S}[\mathbf{x}(t_1) - \mathbf{x}_1]$$

The optimal control law  $u^*(t)=i^*_{sq}(t)$  results from the integration of the canonic system (6).

In order to avoid the classic recursive solution of the MRDE with the well known disadvantages and the positive eigenvalues of the system, the current time 't' goes to ' $t_1$ -t', time remaining until the end of the optimal process, through the adequate conversion of the state coordinates (Gaiceanu, 2004b).

The optimal control law, at any moment 't', is, (Rosu, *et al.*, 1998 a,b; Gaiceanu, 1999; Gaiceanu, 2004 b):

(10) 
$$u^{*}(t) = -\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P}(t_{1}-t)\mathbf{x}(t) + \mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{K}_{1}(t_{1}-t)\mathbf{x}_{1} + \mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{K}_{2}(t_{1}-t)\mathbf{w}(t)$$

in which  $\mathbf{P}(t_1 - t)$  is the solution of the MRDE and the matrices  $\mathbf{K}_1$  and  $\mathbf{K}_2$  are calculated via  $\mathbf{P}(t_1-t)$ .

The optimal control law has three components (Fig.1): the state feedback, the forcing component to achieve the desired state  $\mathbf{x}_1$  and the compensating feed forward of the perturbation  $\mathbf{w}(t)$ .

Obviously, the analytical solution supposes the knowledge of the perturbation  $\mathbf{w}(t)=m_l(t)$ , which could be available by using a load torque estimator, as in (Gaiceanu, 1996; Rosu, *et al.*, 1998c) or a torque sensor.

## 4. THE STRUCTURE OF THE DEVELOPMENT SYSTEM

The optimal control for ac drives has been implemented by means of an experimental test bed (Fig.2). The structure of the development system involves the three-phase induction motor, an electrical brake, the ac-ac converter, the dSPACE board and the current/voltage box of the transducers (Figure 2).



Fig.1. The optimal control system.



Fig.2. The developed experimental test bed

In order to implement the optimal control, the dSPACE board computes the real-time MRDE's solution. The obtained optimal solution is based on the corresponding feedback signals (line currents, phase voltages, IM rotor position and load torque), and on the references (Figure 1). By using the mathematical model of the IM, the dSpace controller delivers the optimal speed profile reference signal for

the Altivar 71 converter (Fig.5). The Altivar71 tracks the optimal speed reference through the implemented rotor field oriented control. The results of the developed AC drive system consist of energy reduction in all system.

### 5. EXPERIMENTAL RESULTS

Maintaining the flux component of the stator current at the constant value, 3,827[A], the optimal control law (8) and the mathematical model (1) have been numerically simulated and implemented for starting period of a 2.2 [kW], 1420 [rpm] 1LA7106-4AA10-Siemens induction motor without and with load torque. The initial conditions of the optimal control problem are:  $t_o=0[s]$ , ( $n_i=0$  [rpm],  $q_i=0$ ). The free final conditions are:  $t_r=0.65[s]$ , ( $n_r=1500$  [rpm],  $q_i=0$ ). Both types of control, i.e. classical and optimal (Figs.3,4), have been implemented on DS1104 controller board.

The electrical drive signals (Figs.3-4), i.e. the line currents ( $i_A$ ,  $i_B$ ,  $i_C$ ), the supplying voltages of the induction motor ( $u_R$ ,  $u_S$ ,  $u_T$ ), the load torque  $M_f$ , and the rotor postion, can be monitored in real time by means of the developed ControlDesk (Figs.3,4). The ControlDesk acts as an interface between the DS1104 controller and PC. The functions of the ControlDesk interface are: monitoring, load torque magnitude control, the type of the real-time control choosing (classical or optimal); real-time energy analyses for both type of controls. The output active and reactive power of the system, the speed, the RMS values of the phase current and voltage, and the output frequency of the inverter are calculated on-line based on the acquisitioned feedback signals (Figs.3-4).

The references of the desired final state  $(x_1)$  are provided by DS1104 through a digital to analog (DAC) channel of the connector panel CP1104 by means of the ControlDesk. Comparison between the classical and optimal system speed references is shown in Fig.5. The similar test has been performed for a braking. In Fig.7 the experimentation platform for the development of the optimal drives (dc and ac) is depicted.



Fig.3. The classical control system results.

The analysis reveals the system efficiency improvement through an important decreasing of the windings dissipation power (Figs3,4) and input power (Fig.6) in optimal control case during drive starting. Therefore, the power balance provides an increasing of the system efficiency, thus improved thermal and capacity conditions are obtained.



Fig.4. The optimal control system results



Fig.5. The AC drive speed references: comparative results.



Fig.6. The AC drive input energy comparison [p.u.]: classical and optimal control system



Fig.7. The experimentation platform for the development of the optimal drives.



Fig.8. The experimentation platform for the development of the optimal drives.

From the Fig.6 it could be noted that: 1) during the AC drive starting the input energy decreases in optimal control case; 2) during the steady state regime both energy consumptions are the same.

## 6. CONCLUSIONS

In this paper the optimal control experimental test bed has been shown.

The dedicated optimal control developed system includes both drive systems: AC and DC.

The developed optimal control test bed is orientated to research purposes.

The real-time optimal control solution is delivered by the dSpace controller interconnected with the Altivar71 converter.

The classical control (consisting of rotor field oriented control with PI controllers) and the optimal one have been implemented by designing an adequate ControlDesk interface. In order to implement the optimal control, the dSPACE board computes the real-time MRDE's solution.

The solution features of the optimal control problem are:

- The nonrecursive solution has been used to avoid the main disadvantages of the recursive solution (that is, the solution can be calculated at the current time, not backward in time; the load torque could have any shape: either linear or nonlinear, in admissible limits due to the fact that solution is computed at each sample time);
- The optimal problem is unconstrained, the magnitude constraints for the control and state can be solved by adequate choice of the weighting matrices **S**, **R** and **Q**;
- According to Bellman's theorem of optimality, the solution obtained for a starting is extended for a braking or a reversing process.

The features of the optimal controller:

- The optimal control law provides about 19 % reduction in electrical losses compared to a standard control design;
- High dynamic performances, without overshoot and the fast compensation of the load torque;
- Smooth dynamic response;
- Due to the negative real eigenvalues, the stable system is obtained;
- For the drives with frequent reversing and high power the solution can be feasible.

The reduction of the energy assures either an increasing of the operational period of the electrical drives components, or the induction motor overload permission.

#### 7. REFERENCES

- Athans, M., Falb, P. L. (1966). *Optimal Control*, Mc Graw-Hill, New York,.
- Cheng, T., F.L.Lewis, M.Abu-Khalaf (2007). A neural network solution for fixed-final time optimal control of nonlinear systems, *Automatica, Journal of IFAC*, Vol.43, Pergamon Press, Inc., March 2007, pp. 482-490
- Deneika, V.S. (2006). Optimal control of the dynamic state of a thin compound plate, *Journal of Cybernetics and Systems Analysis*, Vol.42, Kluwer Academic Publishers, ISSN 1060-0396, pp. 587-608

- Gaiceanu, M. (1996). Optimal Control of DC Drives, Licentiate Thesis, Dunarea de Jos University of Galati
- Gaiceanu, M., Rosu, E., Lungeanu, F and Bivol, I. (1999). The stator voltage optimal space-vector for induction machine, ElectroMotion'99, Patras, Greece, **Vol. I**, 297-302.
- Gaiceanu, M. (2004a). AC-AC Converter System for AC Drives, Second IEE International Conference on Power Electronics, Machines and Drives (PEMD2004), University of Edinburgh, UK, ISBN 0-86341-383-8, vol.2, 31 March – 2 April 2004, pp. 724-729.
- Gaiceanu, M. (2004b). *Reglarea optimala a sistemelor de conversie electromecanica*, Ed. Didactica si Pedagogica, Bucuresti, ISBN 973-30-1877-5, 244 pp.
- Jianqiang, L., Yang, L., F., Zheng, Z., Trillion., Q. (2007). Optimal Efficiency Control of Linear Induction Motor Drive for Linear Metro, 2nd IEEE Conference on Industrial Applications, 2007, ICIEA 2007, 23-25 May 2007, pp. 1981-1985, ISBN 978-1-4244-0737-8
- Leonhard, W. (1996). *Control of Electrical Drives*, Springer-Verlag, Berlin
- Matinfar, M., and K.Hashtrudi-Zaad (2005).
  Optimization-based Robot Compliance Control: Geometric and Linear Quadratic Approaches, International *Journal of Robotics Research*, ISSN 0278-3649, Sage Publications, Inc., Vol.24, pp. 645-656, August 2005
- Rosu, E., Gaiceanu, M., Bivol, I (1998a). Optimal Control for AC Drives Supplied from PWM Voltage Source Inverter, IEEE Optim '98, 6th International Conference on Optimization of Electrical and Electronic, Equipments, Brasov, Romania, 427-432.
- Rosu, E., Gaiceanu, M., Bivol, I.(1998b). Optimal Control Strategy for AC Drives.IEEE- PEMC '98, 8th International Power Electronics & Motion Control Conference, Prague, Czech Republic, 4.160-4.165
- Rosu, E., Gaiceanu, M., Bivol, I (1998c). Load Torque Estimation for AC Motors, CNAE '98, The 9th Symposium on Electrical Drives, Craiova, Romania, 1998, pp.221-224.
- Su, C.T., and C.L. Chiang (2004). Optimal Position/Speed Control of Induction Motor Using Improved Genetic Algorithm and Fuzzy Phase Plane Controller, *Journal of Control and Intelligent Systems*, ISSN 1480-1752, Vol.32,
- Tamimi, J.M.Kh., and H.M. Jaddu (2006). Optimal vector control of three-phase induction machine, International Association of Science and Technology for Development, Proceedings of the 25<sup>th</sup> IASTED International Conference on Modeling, Identification and Control, Lanzarote, Spain, Acta Press, CA USA, ISSN 0-88986-549-3 pp. 92-96

- Veerachary M. (2002). Optimal control strategy for a current source inverter fed induction motor, *Computers and Electrical Engineering*, Vol.28, No.4, July 2002, pp.255-267
- Zhongke, S. (2004). Optimal fuzzy control for nonlinear system and application to AC motor control, Power Electronics and Motion Control Conference, 2004, IPEMC 2004, The 4<sup>th</sup> International, Vol.3, 14-16Aug.2004, pp.1331-1334

Acknowledgments - The results have been obtained through the Research Program of Excellence, CEEX-MENER, No.603/10/2005, Romania