FEEDBACK LINEARIZATION CONTROL OF WIND POWER SYSTEMS

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Abstract: This paper focuses on the development of a feedback linearization control for a variable speed fixed pitch wind turbine driving a permanent magnet synchronous generator. The power system is considered to operate on an insular grid. The feedback linearization controller aims to maximize the energy captured from the wind, for varying wind speeds. Numerical simulation results are presented to demonstrate the effectiveness of the feedback linearization controller.

Keywords: Wind generation; Windmills; Feedback linearization; Permanent magnet; Synchronous generator;

1. INTRODUCTION

In the last years the use of wind power systems supplying with electricity isolated communities has known an important increase. A common configuration for such wind power systems uses a variable speed fixed pitch horizontal axis wind turbine (HAWT) driving a permanent magnet synchronous generator (PMSG) connected to the local grid through power electronics as shown in Figure 1.

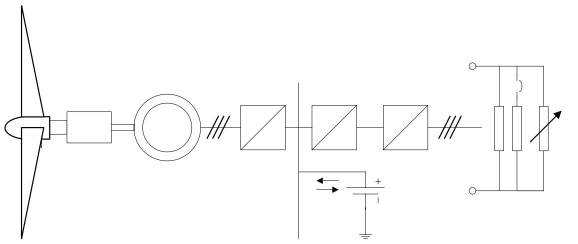

Fig.1. The HAWT – PMSG power system

In such variable speed wind power systems the control problem consists mainly in maximizing the energy captured from the wind for varying wind speeds.

Nonlinear control techniques (feedback linearization) have been used to design control structures for power systems using hidraulic turbines driving synchronous generators with excitation connected to the infinite bus. [Chapman, *et al.*, 1993, Mielczarski, *et al.*, 1994, Akhrif, *et. al.*, 1999] but there are no literature references showing the use of feedback linearization technique to design a control system for HAWT – PMSG power systems connected either to the infinite or to an insulated grid.

In this paper the feedback linearization technique is used to design a control structure for the highly nonlinear HAWT – PMSG power system connected to an insulated grid.

The paper is organized as follows. The wind power system modeling is presented in next section. Section 3 presents the design of the feedback linearization controller. The numerical simulation results that show the viability of the feedback linearization control design are presented in Section 5. Some concluding remarks end this paper.

2. SYSTEM MODELING

The kinetic energy of the moving air masses – wind – is captured by a turning device – turbine rotor – and transformed into mechanical energy – turbine shaft – and further into electrical energy – electrical generator. If the wind energy is fully captured by the turbine rotor, the total power would be

$P_t = \frac{1}{2} \cdot \pi \cdot \rho \cdot a \cdot v^3$, where ρ is the air density, a is the section area of the wind turbine and v is the wind speed. In reality the wind turbine harvest from the wind a mechanical power, P_w, smaller than the total power, P_t, due to the non-zero wind speed behind the rotor. The expressions of P_w is obtained according to Rankine – Froude theory of propellers in incompressible fluids, reconsidered by Betz in 1926 [Burton, *et al.*, 2001]:

$$(1)\ P_v = \frac{1}{2} \cdot \rho \cdot a \cdot v^3 \cdot C_p$$

where C_p is the power coefficient defining the aerodynamic efficiency of the wind turbine rotor, and is a function of the tip speed ratio, λ. The tip speed is defined as the ratio between the peripheral speed of the blades and the wind speed:

$$(2)\ \lambda = \frac{R \cdot \Omega}{v}$$

where Ω is the rotational speed of the blades (the rotational speed of the low-speed shaft) and R is the blade length.

The typical performance curve for a horizontal axis wind turbine is given in Figure 2.

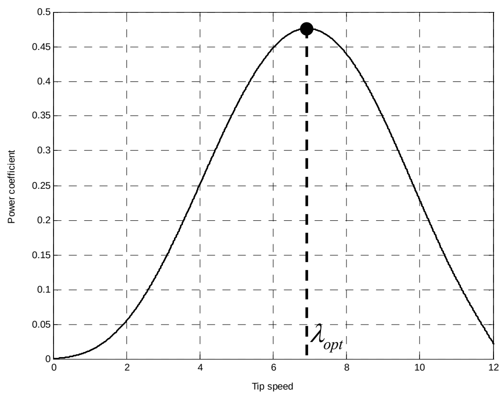

Fig.2. Power coefficient versus tip speed

It presents a maximum for a well-determined tip speed, denoted by λ_{opt}.

The power characteristics for different wind speeds, are given in Figure 2. For every wind speed they have a maximum. All these maxima determine a so-called *Optimal Regimes Characteristic* (ORC), as shown in Figure 3.

In order to maximize the power extracted from the wind, the tip speed ratio should be kept around it's optimal value.

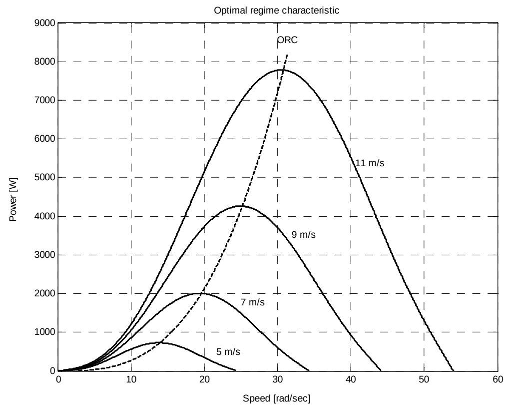

Fig.3. The optimal regime characteristics

2.1. Wind turbine model

The wind turbine provides the shaft's mechanical torque, according to the wind torque expression:

$$(3)\ T_w = \frac{1}{2} \rho \cdot \pi \cdot C_T(\lambda) \cdot R^3 \cdot v^2$$

where R is the blade radius and $C_T(\lambda)$ is the torque coefficient, defined as the fraction between the power coefficient and the tip speed:

$$(4)\ C_T(\lambda) = \frac{C_p(\lambda)}{\lambda}$$

The torque coefficient is an designed parameter and it is usually provided by the manufacturers of the wind turbine. For the wind turbine considered in this paper, the torque coefficient is modeled as a 6th order polynomial regression [Nichita, 1995]:

$$(5)\ C_T(\lambda) = a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3 + a_4\lambda^4 + a_5\lambda^5 + a_6\lambda^6$$

The torque coefficient described by eq. (5) is presented in Figure 4.

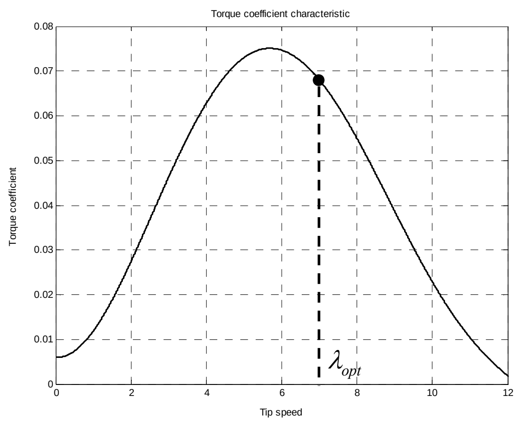

Fig.4. The torque coefficient versus tip speed

The operating point corresponding to optimal tip speed $(\lambda_{opt} = 7)$ lies on the descending portion of the torque coefficient, as it can be seen in Figure 4.

The gear box is considered to be rigid and without dynamic, with a gear ratio $i = 10$.

2.2. PMSG model

The PMSG is modeled in the dq frame, discarding the zero component, and it is connected to the infinite power bus:

$$(6)\quad \begin{aligned} u_d &= -Ri_d - L_d \frac{\mathrm{d}i_d}{\mathrm{d}t} + pL_q\omega i_q \\ u_q &= -Ri_q - L_q \frac{\mathrm{d}i_q}{\mathrm{d}t} - p\left(L_d i_d - \Phi_m\right)\omega \\ J\frac{\mathrm{d}\omega}{\mathrm{d}t} &= T_w - p\left[\Phi_m i_q - \left(L_d - L_q\right) i_d i_q\right] \end{aligned}$$

where R - the rotor resistance, p – the pole pair number, L_d, L_q - the rotor inductances in the dq axes, J – the moment of inertia and Φ_m - the permanent magnet flux. The dq axes voltages, u_d and u_q, are the input variables.

When connected to a local grid, u_d and u_q become output variables, so the PMSG should include the local grid equations. The voltages on the dq axes, described by the local grid equations, are:

$$(7)\quad \begin{bmatrix} u_d \\ u_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + L_s \frac{\mathrm{d}}{\mathrm{d}t} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + X \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

where

$$(8)\quad X = T^{-1} \cdot \frac{\mathrm{d}T^{-1}}{\mathrm{d}t} = \begin{bmatrix} o & \omega L_s \\ -\omega L_s & 0 \end{bmatrix}$$

with R_s - the load resistance, L_s - the load inductance and T - the Park transformation matrix [Leonhard, 1986].

We replace eq. (7) in eq. (6):

$$(9)\quad \begin{aligned} &\left(L_d + L_s\right)\frac{\mathrm{d}i_d}{\mathrm{d}t} = -\left(R + R_s\right) i_d + p\left(L_q + L_s\right) i_q \omega \\ &\left(L_q + L_s\right)\frac{\mathrm{d}i_q}{\mathrm{d}t} = -\left(R + R_s\right) i_q - p\left(L_d + L_s\right) i_d \omega + p\Phi_m \omega \\ &J\frac{\mathrm{d}\Omega}{\mathrm{d}t} = T_w - T_{em} = T_w - p\left[\left(L_d - L_q\right) i_d i_q - \Phi_m i_q\right] \end{aligned}$$

The wind torque expression in eq. (5) provides the torque coefficient characteristic for all operating regimes. On the other hand, the controller aims to maximize the energy captured from wind, thus the region of interest is the descending pant of the torque coefficient characteristic (Figure 4), the region for tip speed $\lambda \geq 6$.

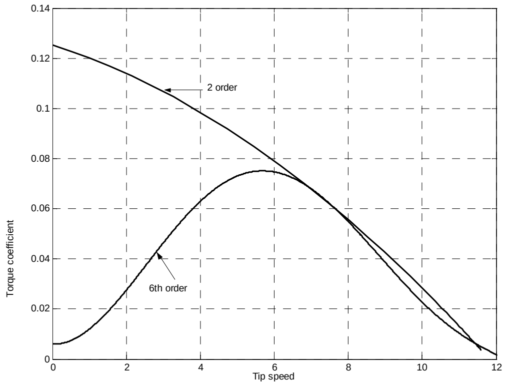

Fig.5. The 2 torque coefficient characteristics

In the feedback linearization controller design a second order polynomial regression was considered, as shown in Figure 5.

The PMSG model, with the permanent magnets mounted on the rotor surface $(L_d = L_q)$, can be expressed in the state variables form:

$$(10)\quad \begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} x = f(x) + g(x) \cdot u \\ &y = h(x) \end{aligned}$$

or

$$(11)\quad \begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} a_1 x_1 + a_2 x_2 x_3 \\ b_1 x_2 + b_2 x_1 x_3 + b_3 x_3 \\ d_1 v^2 + d_2 v x_3 + d_3 x_3^2 + d_4 x_2 \end{bmatrix} + \\ &+ \begin{bmatrix} a_3 \cdot x_1 \\ b_4 \cdot x_2 \\ 0 \end{bmatrix} \cdot R_s \end{aligned}$$

where

$$(12)\quad \begin{aligned} &a_1 = -\frac{R}{L_d + L_s}; a_2 = p\frac{L_q - L_s}{L_d + L_s}; a_3 = -\frac{1}{L_d + L_s}; \\ &b_1 = -\frac{R}{L_q + L_s}; b_2 = -p\frac{L_d + L_s}{L_q + L_s}; b_3 = p\Phi_m; \\ &b_4 = \frac{1}{L_q + L_s}; d_1 = \frac{1}{2J}\pi\rho R_t^3 c_0; \\ &d_2 = \frac{1}{2J}\pi\rho R_t^4 c_1; d_3 = \frac{1}{2J}\pi\rho R_t^5 c_2 \end{aligned}$$

c_0, c_1 and c_2 are the parameters of the torque coefficient (given in Appendix). The dynamic of the power electronics – rectifier, chopper and inverter – being significantly more rapid than the wind turbine – PMSG dynamic, was neglected. Thus, the input variable R_s represents the equivalent load resistance at generator's terminals.

3. FEEDBACK LINEARIZATION CONTROLLER DESIGN

We consider the nonlinear state variable wind power system model given in eq. (10):

$$(13)\quad f(x)=\begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix}=\begin{bmatrix} a_1x_1+a_2x_2x_3 \\ b_1x_2+b_2x_1x_3+b_3x_3 \\ d_1v^2+d_2vx_3+d_3x_3^2-d_4x_2 \end{bmatrix}$$
$$g=\begin{bmatrix} a_3x_1 & b_3x_2 & 0 \end{bmatrix}^T;$$

and:

$$x=\begin{bmatrix} i_d & i_q & \omega \end{bmatrix}^T;$$
$$(14)\quad u=R_s$$
$$h(x)=x_3=\omega$$

In order to find the relative degree of the system, we compute the Lie derivatives [Isidori, 1989]:

$$(15)\quad \begin{aligned} L_f h(x) &= d_1v^2+d_2vx_3+d_3x_3^2-d_4x_2 \\ L_g L_f h(x) &= -d_4a_3x_2 \neq 0 \end{aligned}$$

The relative degree of the system is $r=2$ and is smaller than the system order. In this case, an exact feedback linearization is not possible [Isidori, 1989], but we can still achieve a partial feedback linearization, resulting in the decomposition of the wind power system into a second order linear subsystem, responsible for the input – output behavior, and a fist order nonlinear subsystem, representing an internal dynamic that does not affect the input – output behavior.

We introduce the change of coordinates:

$$(16)\quad \begin{aligned} z_1 &= h(x)=x_3 \\ z_2 &= L_f h(x)=d_1v^2+d_2vx_3+d_3x_3^2-d_4x_2 \\ z_3 &= a_3\frac{x_1}{x_2} \end{aligned}$$

In the new coordinates, the system dynamics are:

$$(17)\quad \begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= (2d_1v+d_2z_1)\dot{v}+(d_2v+2d_3z_1)z_2-(b_1+b_4\cdot R_s+b_2a_3z_1z_3)\cdot A \\ \dot{z}_3 &= a_2z_1+(a_1a_3-b_1)z_3-b_3a_3z_1z_3^2 \end{aligned}$$

with

$$(18)\quad A=d_1v^2+d_2vz_1+d_3z_1^2-z_2$$

The control input will be:

$$(19)\quad u=\frac{1}{L_gL_fh(x)}\left(-L_f^2h(x)+v\right)$$

with

$$(20)\quad \begin{aligned} L_f^2h(x) &= -d_4\cdot f_2(x)+(d_2v+2d_3x_3)\cdot f_3(x) \\ L_gL_fh(x) &= -d_4a_3x_2 \end{aligned}$$

where $f_2(x)$ and $f_3(x)$ are given in eq. (13).

The command is composed by a state variable feedback linearization, given by the Lie derivatives $L_f^2h(x)$ and $L_gL_fh(x)$, and a control input v which allows us to impose the desired dynamic to the input – output linear subsystem.

The control input v is a state variable feedback command [Ceanga, *et al.*, 2001] with an integrator to assure zero error in stationary regime. The state variable feedback command scheme is presented in Figure 6.

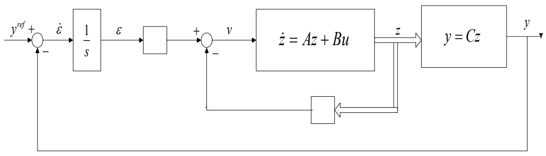

Fig.6. State variable feedback command scheme

The input – output linear subsystem is:

$$(21)\quad \begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \end{aligned}$$

The command is:

$$(22)\quad u=-\begin{bmatrix} k_1 & k_2 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}+k_I\varepsilon$$

where

$$(23)\quad \varepsilon=y^{ref}-y=y^{ref}-\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

is the reference tracking error.

The parameters k_1, k_2 and k_I were calculated by pole allocation technique [Åström, 1990] imposing a pair of dominant poles with $\omega_0=20$ and $\zeta=0.9$:

$$(24)\quad k_1=4000;\; k_2=136;\; k_I=40000$$

4. SIMULATION RESULTS

The feedback linearization control scheme for wind power systems was implemented and numerically simulated in Matlab/SIMULINK® (Figure 7).

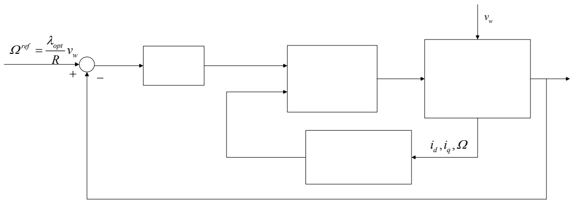

Fig.7. Feedback linearization control block scheme

The aim of the feedback linearization control is to maximize the energy captured from wind, namely maintaining the tip speed around its optimal value $(\lambda_{opt} = 7)$. The speed reference Ω^{ref} is calculated according to the measured wind speed v_w:

$$(25)\quad \Omega^{ref} = \frac{\lambda_{opt}}{R} v_w$$

where *R* is the blade radius.

In order to validate the feedback linearization speed tracking controller, numerical simulations with a determinist probe signal were conducted. The results are presented in Figure 8, where the reference speed is the dotted line and the shaft speed is the solid line.

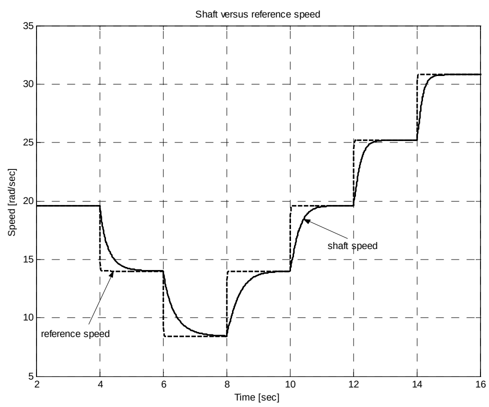

Fig.8. Shaft speed versus reference speed

The feedback linearization control scheme has good tracking capabilities and assures zero tracking error in stationary regime.

The signal presented in Figure 9 represents a realistic wind profile, modeled as a non-stationary stochastic process [Cutululis, *et al*., 2001] and used as the input to the simulation.

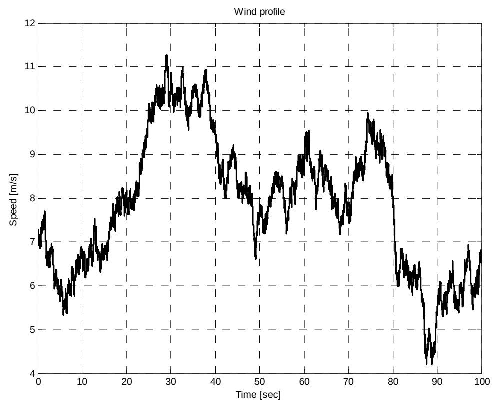

Fig.9. The wind profile

The wind profile covers a speed range between $4 \div 11 \text{m/s}$, which represents the range between wind turbine starting speed and nominal speed. This is the speed range that covers most of the wind turbine's operating time.

The effectiveness of the feedback linearization control scheme can be seen in Figure 10. The controller ensures maximum energy caption from wind, for wind speed varying from 4 to 11 m/s. The tip speed is maintained around its optimal value $(\lambda_{opt} = 7)$ with dynamical errors due to the turbulence component of wind speed of maximum ± 1.

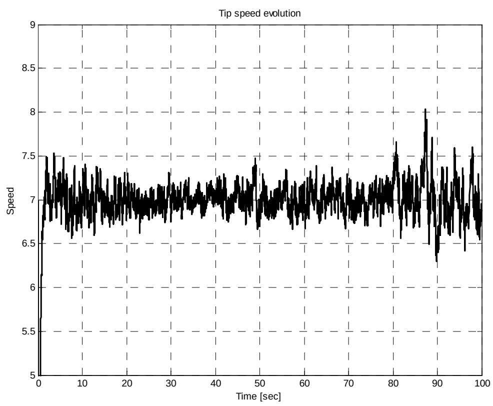

Fig.10. Tip speed evolution

Figure 11 shows the dynamic evolution of the operating point around the ORC. The feedback linearization controller maintains the operating point around the ORC for wind speeds varying from 4 to 11 m/s.

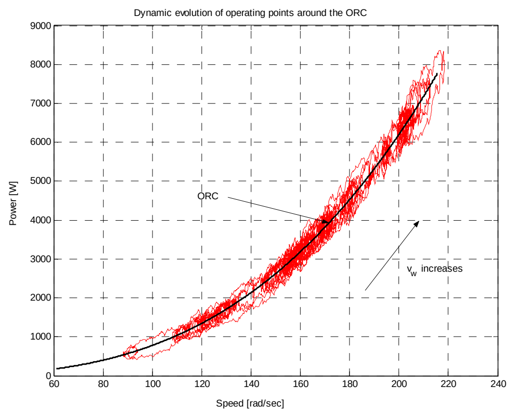

Fig.11. Dynamic evolution of operating point around the ORC

5. CONCLUDING REMARKS

The synthesis of a feedback linearization control scheme for insulated wind power systems is presented in this paper. The wind power system uses a variable speed fixed pitch horizontal axis wind turbine with a permanent magnet synchronous generator connected to a local grid. A state variable model represented the whole system, with input control variable the equivalent load resistance at generator's terminals.

With a state variable transformation, the system is decomposed into a second order linear subsystem, responsible for the input – output behavior, and a first order nonlinear subsystem representing the internal dynamic that does not affect the input – output behavior. The command is synthesized as a state variable feedback linearization and a control input that imposes the dynamic of the linear subsystem. Numerical simulations conducted in Matlab/SIMULINK® show the effectiveness of the feedback linearization scheme.

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7. APPENDIX

PMSG parameters:

$R = 3{,}3\,\Omega$;
$L_d = L_q = 41.56e-3\,\mathrm{H}$;
$J = 0.042\,\mathrm{kgm}^2$;
$\Phi_m = 0.4382\,\mathrm{Wb}$;
$p = 3$;

Wind turbine parameters:

$\rho = 1.25\,\mathrm{kg/m}^3$;
$R_t = 2.5\,\mathrm{m}$;
$J_T = 0.005\,\mathrm{kgm}^2$
$c_0 = 0.1253;\ c_1 = -0.0047;\ c_2 = -0.0005;$