DECOMPOSITION METHODS TO OPTIMISE THE STRUCTURE AND PARAMETERS OF TRANSFER LINE WITH PARALLEL AND SEQUENTIAL MACHINING

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Abstract: The paper is devoted to a preliminary design problem of paced automatic transfer lines. At this design stage, the following decisions are to be made: the partitioning of the given set of operations into subsets performed by one spindle head; the grouping of the spindle heads into workstations, the choice of operating modes for each spindle head. The aim is to minimise the line life cycle cost per part under the given productivity and technological constraints. The paper focuses on a mathematical model of the problem and methods to solve it. The model is formulated in terms of mixed (discrete and non-linear) programming. For solving the optimisation problem, a special decomposition scheme is proposed, which is based on parametric decomposition technique as well as on Branch-and-Bound and multi-criteria shortest path algorithms.

Keywords: Production systems, Optimisation, Decomposition methods, Graph theory.

1. INTRODUCTION

In this paper, the preliminary design stage of transfer lines with multi-head machines is considered. The lines represent "hard automation" for the mass production of a single type of parts in mechanical industry (Groover, 1987; Hitomi, 1996). The parts machined are characterised by a great number of operations and surfaces to be machined, by several types of machining. The lines are designed for a long exploitation time and need high investments, but provide high productivity with relatively small maintenance cost.

The investigated transfer line consists of several workstations (stations) in series with a common transfer mechanism without buffers and with a common control system. The movements of a part between stations are synchronised. In such lines all the stations perform their operations simultaneously, and failure of one station (or necessity to change tools) results in stoppage of the line. A set of operation executed at one station can be partitioned into several subsets, and all operations of each such subset are performed simultaneously by the same spindle head. Spindle heads of a station work sequentially according to their numeration. Parameters of each station are defined by the set of operations performed as well as by the number of used spindle heads. Parameters of the spindle head and its operating time depend on its operation set and on accepted operating modes.

In the preliminary design stage, the following decisions must be made: the partitioning of the given set of operations into subsets which are performed by one spindle head; the grouping of spindle heads at stations, choice of operating modes for each spindle head. The design decision defines the line life cycle cost per part. The quality of accepted decision must be rather high, because even slight improvement of the design decision allows getting a substantial economic benefit (effect of large scale).

In this paper, we focus on some mathematical aspects of the preliminary line design stage. Several other mathematical aspects of the considered design problem were investigated in (Levin and Tanaev, 1978; Guschinsky and Levin, 1990; Dolgui *et al.*, 1999a,b, 2002a,b). Close problems of assembly line balancing are discussed in (Arcus, 1966; Baybars, 1986; Talbot *et al.,* 1986; Jonson, 1988; Ghosh and Gagnon, 1989; Scholl, 1999; Rekiek *et al.*, 2003). Problems of designing assembly line with equipment selection are investigated in (Graves and Lamar, 1983; Pinto *et al.*, 1983; Graves and Holmes, 1988; Rubinovitz and Bukchin, 1993; Bukchin and Tzur, 2000).

2. PROBLEM STATEMENT

2.1. The transfer lines: case of study

We consider a case when the operating time of a spindle head defined by its length of working stroke and the feed per minute. In one's turn, both parameters depend on executed operations and accepted operating modes for each of them. Choice of operating modes is very important. Great modes result in small operating times, but demand more frequent tool replacement, etc. We assume operating modes (the feed per revolution and the cutting speed) for each operation are uniquely defined by the accepted feed per minute of the corresponding spindle head.

The investigated problem is to optimise:

a) the number m of stations;
b) the partitioning of the given set $\mathbf{N}$ of operations into subsets N_k, k=1,...,m, (all operations of N_k are executed at k-th station);
c) the numbers n_k of spindle heads at k-th station, k=1,...,m;
d) the partitioning of the sets N_k into subsets N_{kj}, j=1,...,n_k, (all operations of N_{kj} are executed simultaneously by kj-th spindle head), k=1,...,m;
e) the feed per minute X_{kj}, j=1,..., n_k, k=1,...,m.

2.2 Optimisation criterion

Let $P=\langle P_1 ,...,P_k ,...,P_m\rangle$ be a design decision with stations $P_k=(P_{k1} ,...,P_{kj} ,..., P_{kn_k})$ and with spindle heads $P_{kj}=(N_{kj},X_{kj})$. An estimation $\Theta=\Theta(P)$ of the line life cycle cost per part will be used to choose the best design decision. The estimation involves the cycle time, the labour cost (wages of the operators, including overheads), the line acquisition and the installation cost, time and cost for set-up, repair and maintenance, tools set-up time and cost, area occupied by the system, planned output (production volume).

The decomposition methods, which are developed in this paper, are based on the specificity of the structure of the life cycle cost function. The following dependencies of the cost components on unknown design parameters are considered:

(i) The execution time $\tau(N_{kj},X_{kj})$ of the set N_{kj} of operations by the kj-th spindle head for the feed per minute X_{kj} is equal to:

$$\tau(N_{kj},X_{kj})= \tau'+max\{ l(p) \mid p \in N_{kj} \}/X_{kj} ,$$

where τ' is a constant, which characterises idle stroke time, and $l(p)$ is the given length of working stroke for operation $p \in \mathbf{N}$.

The executiontime of all the sets N_{kj} of operations, j=1,2,...,n_k, at k-th station is equal to $\sum_{j=1}^{n_k} \tau(N_{kj},X_{kj})$. Then the line cycle time is equal to:

$$t(P)= \tau''+max\{ \sum_{j=1}^{n_k} \tau(N_{kj},X_{kj}) \mid k=1,2,...,m\},$$

where τ'' is a constant, which characterises an additional time.

(ii) The relative (per part) time for set-up, repair and maintenance can be estimated as the sum of the same characteristics for stations and spindle heads. In turn, each of these time characteristics can be considered as the sum of two items, one of them is proportional to the cycle time, and the second does not depend on it. The similar assumptions are made about relative (per one part) cost of line repair and maintenance.

(iii) The variable component (which depends on design decision) of the line acquisition cost can be also considered as the sum of the corresponding costs for stations and spindle heads. These costs involve also the costs of consumables as well as the costs for installation of the transfer line. By analogue, area occupied by the system can be considered as the sum of station areas.

(iv) If ranges $[s_1(p),s_2(p)]$ and $[\delta_1(p),\delta_2(p)]$ of respectively feasible values of feeds per revolution and spindle speeds (cutting speeds) are known and each operation $p \in \mathbf{N}$ is performed by a separate tool of corresponding spindle head, then

$$X_{kj}(N_{kj})= [\underline{X}(N_{kj}),\ \overline{X}(N_{kj})]$$

is the set of feasible values of feeds per minute for the *kj*-th spindle head, where

$$\underline{X}(N) = max\{x_1(p) | p \in N\},$$

$$\overline{X}(N) = min\{x_2(p) | p \in N\},$$

$$x_r(p) = s_r(p)\delta_r(p),\ r=1,2.$$

For a fixed value of feed per minute $x \in X_{kj}(N_{kj})$ and any $p \in N_{kj}$, the feed per revolution is supposed equal to $s(p,x) = min[s_2(p), x/\delta_1(p)]$ and the spindle speed is equal to $\delta(p,x) = x/s(p,x)$.

The relative (per one part) time I_1 of tools set-up and the relative tool cost I_2 for the transfer line can be considered as the sums of the characteristics of the same name for each separate tool, which can be estimated as:

$$i_q(p,x) = i_{q0}(p)(s(p,x)/s_0(p))^{\gamma(p)}(\delta(p,x)/\delta_0(p))^{\eta(p)},\ q=1,2,$$

where $i_{q0}(p)$, $s_0(p)$, $\delta_0(p)$, $\gamma(p)$, and $\eta(p)$ are given constants (usually $\eta(p) > \gamma(p) > 0$); $i_{10}(p)$ and $i_{20}(p)$ are the tool set-up time and the tool cost for the operation p for the fixed values $s_0(p)$ and $\delta_0(p)$ of the cutting conditions. Therefore, it is assumed that

$$(1)\ I(P) = \alpha I_1(P) + I_2(P) = \sum_{k=1}^{m}\sum_{j=1}^{n_k} I(N_{kj}, X_{kj}),$$

where

$$I(N_{kj}, X_{kj}) = \sum_{p \in N_{kj}} (\alpha i_1(p, X_{kj}) + i_2(p, X_{kj})).$$

Under these assumptions, the function $\Theta(P)$ has the following structure:

$$\Theta(P) = t(P) \times \left[\alpha + \sum_{k=1}^{m}\left(c_1(N_k, n_k) + \sum_{j=1}^{n_k} c_2(N_{kj})\right)\right]$$

$$(2)\ + \sum_{k=1}^{m}\left(c_3(N_k, n_k) + \sum_{j=1}^{n_k} (c_4(N_{kj}) + I(N_{kj}, X_{kj}))\right),$$

where α is the labour cost, $c_r(.)$ include cost components of a station (r=1,3) and a spindle head (r=2,4), respectively.

2.3. Technological constraints

The following constructional and technological factors are taken into consideration:

(a) The given output (production volume) and the real value of line operating time by period (for example, one year), the forecast value of down times (line stoppages), and other factors allow to evaluate the maximum permissible value of the line cycle time. It is assumed that the given line productivity is provided, if the line cycle time $t(P)$ does not exceed the maximum value of the cycle time.

(b) A number of known technological factors (such as fixed sequences of part elements machining by a set of tools, mutual position of surfaces to be machined, the necessity of intermediate transitions during machining, etc.) determine a order relation on the set **N**, which defines possible sequences of operations.

(c) The required precision (tolerance) of mutual position of machined part elements as well as a number of additional factors implies the necessity to perform some groups of operations from **N** at the same station or even by the same spindle head for each group.

(d) At the same time, the possibility to perform some group of operations from **N** at the same station or by the same spindle head is also defined by a number of constructional and technological constraints, for instance, mutual influence of combining operations, possibility of tool location in spindle head, etc.

3. MATHEMATICAL MODEL

To formulate a mathematical model of the considered design problem, let us introduce the following additional notations:

$\bar{t}$ is the maximum admissible value of the cycle time;

(**N**,L) is a digraph, which specifies partial order relation on the set **N**;

$M(p)$ specifies the assignment of the operation $p \in$ N to a station and to a spindle head according to design decision P, i.e. if $p \in N_{kj}$, then $M(p) = (k,j)$;

E^b (E^s) is the set of groups of operations from **N**, which must be performed by the same spindle head (at the same station) for each group;

H^b (H^s) is the set of groups of operations from **N**, which are not to be performed by the same spindle head (at the same station) for each group;

m_0 (n_0) is the maximum admissible number of stations (of spindle heads at one station).

Then the mathematical model of the considered problem can be formulated as follows:

$$(3)\ \Theta(P) \rightarrow min,$$

subject to :

$$(4)\ t(P) \le \bar{t},$$

$$(5)\ \bigcup_{k=1}^{m}\bigcup_{j=1}^{n_k} N_{kj} = \mathbf{N},$$

$$N_{k'j'} \cap N_{k''j''} = \varnothing,\ k',k''=1,...,m,\ j'=1,...,n_{k'},$$

$$(6)\ j''=1,...,n_{k''},\ (k',j') \ne (k'',j''),$$

$$(7)\ M(p') \prec M(p''),\ (p',p'') \in L,$$

$$(8)\ N_k \cap e \in \{\varnothing, e\},\ e \in E^s,\ k=1,...,m,$$

$$(9)\ N_{kj} \cap e \in \{\varnothing, e\},\ e \in E^b,\ k=1,...,m,\ j=1,...n_k,$$

(10) $e \not\subset N_k$, $e \in H^s$, $k=1,...,m$,

(11) $e \not\subset N_{kj}$, $e \in H^b$, $k=1,...,m$, $j=1,...,n_k$,

(12) $X_{kj} \in X(N_{kj}) \neq \varnothing$, $k=1,...,m$, $j=1,...,n_k$,

(13) $m=m(P) \leq m_0$,

(14) $n_k = n_k(P) \leq n_0$, $k=1,...,m$,

where $(k',l') \prec (k'',l'')$, if $k' < k''$, or $k'=k''$ and $l' \leq l''$.

This problem has a number of essential differences from the above mentioned line balancing problem. We note only the following:

- all operations N_{kj} of each kj- th spindle head are performed simultaneously, and subsets N_{kj} are not given in advance;
- operating time for kj-th spindle head is determined by set of operations N_{kj} and accepted cutting modes X_{kj};
- there are constraints on necessity and possibility of grouping operations into subsets N_k and N_{kj};
- the objective function $\Theta(P)$ takes into account not only the number of the stations but also other factors.

For solving this problem, specific methods are needed, because the known methods of lines balancing cannot be used directly.

4. SOLUTION METHOD

4.1. Graph approach

The optimisation problem (3-14) may be reduced to the problem of finding a shortest constrained path in the following digraph.

Let $\mathbf{P}$ be a set of collections

$$P=<((N_{11},X_{11}),\cdots,(N_{1n_1},X_{1n_1})),\cdots,$$
$$((N_{m1},X_{m1}),\cdots,(N_{mn_m},X_{mn_m}))>$$

satisfying the constraints (5-12), and $v_{kl} = \bigcup_{r=1}^{k-1} N_r \bigcup \bigcup_{j=1}^{l} N_{kj}$ be the state of part after machining it by *l-th* spindle head of *k-th* station, V be the set of all states of part for all $P \in \mathbf{P}$ (including also the state $v_0=\varnothing$).

The digraph $G=(V,D)$ is constructed in such a way that $(v',v'') \in D$ if and only if $v' \subset v''$ and $N'' = v'' \setminus v'$ satisfies the conditions that follow from constraints (7), (9), (11) and (12):

- For any $(p',p'') \in L$ the condition $p'' \in N''$ implies $p' \in v''$;
- $N'' \cap e \in \{\varnothing, e\}$ for any $e \in E^b$;
- $e \not\subset N_{kj}$, for any $e \in H^b$;
- $X(N'') \neq \varnothing$.

A set $\Gamma_{v'v''}=\Lambda_{v'v''} \times X(N'')$ is assigned to each arc (v',v''). The set $\Lambda_{v'v''}$ can contain either the element 0 or the elements 0 and 1. The second case holds on if and only if $e \cap v''=\varnothing$ or $e \subseteq v''$ for all $e \in E^s$. The set V is partitioned into two subsets $\overline{V}$ and $V \setminus \overline{V}$ with the following properties: for any $(w,v) \in D$ set $\Lambda_{wv}=\{0,1\}$ if $v \in \overline{V}$ and $\Lambda_{wv}=\{0\}$ otherwise.

The graph G is acyclic. It is supposed that the vertices v_i of the set V are enumerated in order of increasing their rang, $i=0,\ldots,M$ ($v_M=\mathbf{N}$).

Each design decision $P \in \mathbf{P}$ can be associated with a parameterised path $z(P)=((v_0=u_0,...,u_{j-1}, u_j,...,u_{l(z)}=v_M), (\gamma_1,..., \gamma_j,..., \gamma_{l(z)}))$ in the digraph G from the vertex v_0 to the vertex v_M with appropriate values $\gamma_j = (\lambda_j, X_j) \in \Gamma_{u_{j-1}u_j}$. The parameter λ_j is equal to 1, if the set $N'' = u_j \setminus u_{j-1}$ is performed by the last spindle head at the corresponding station.

In the parameterised path $z=((v_0=u_0,...,u_{j-1}, u_j,...,u_{l(x)} = v_M), (\gamma_1,..., \gamma_j,..., \gamma_{l(z)}))$, a sequence $j_1,j_2,...,j_{m(z)}=l(z)$ of indices j_r can be selected in which the parameters $\lambda_{j_r}=1$, $r=1,...,m(z)$.

Let $n_r(z)=j_r - j_{r-1}$, $r=1,...,m(z)$, where $j_0=0$. Then parameterised path z in the digraph G defines a decision $P(z)$ satisfying constraints (5)-(9) and (11)-(12), but maybe does not respect constraint (10). For each z the following functions are defined:

$$f^1(z) = \max\{\textstyle\sum_{i=j_{r-1}+1}^{j_r} \tau(u_i \setminus u_{i-1}, X_i) \mid$$
$$\mid r=1,...,m(z)\} + \tau'';$$

$$f^2(z) = \alpha_1 + \textstyle\sum_{r=1}^{m(z)} (c_1(u_{j_r} \setminus u_{j_{r-1}}, n_r(z)) +$$
$$+ \textstyle\sum_{i=j_{r-1}+1}^{j_r} c_2(u_i \setminus u_{i-1}, X_i));$$

$$f^3(z) = \textstyle\sum_{r=1}^{m(z)} (c_3(u_{j_r} \setminus u_{j_{r-1}}, n_r(z)) +$$
$$+ \textstyle\sum_{i=j_{r-1}+1}^{j_r} c_4(u_i \setminus u_{i-1}, X_i));$$

$$g(z) = f^1(z) * f^2(z) + f^3(z).$$

Let Z be the set of all parameterised paths in the graph G from v_0 to v_M. Then the problem (3-14) is equivalent to the following problem (hereinafter problem A) of finding the parameterised path z:

(15) $g(z) \to min$,

subject to:

(16) $z \in Z$;

(17) $f^1(z) \leq \bar{t}$;

(18) $e \not\subset (u_{j_r} - u_{j_{r-1}}), e \in H^s$, $r=1,...,m(z)$;

(19) $m(z) \le m_0$;

(20) $n_r(z) \le n_0$, $r=1,\ldots,m(z)$.

The problem (15)-(16) was investigated in (Guschinsky *et al.*, 1990). To solve this problem, a special decomposition scheme was proposed. Here, this scheme is developed to solve the problem A.

Let *Z* be the set of parameterised paths from Z, which satisfy the constraints (18)-(20), $\underline{t}$ be the minimal value of function $f^1(z)$ on the set *Z*, and $\underline{t} \le \bar{t}$ (otherwise there is no solution of problem A). Then a solution of problem A can be obtained using the following two-level decomposition scheme (see Fig.1).

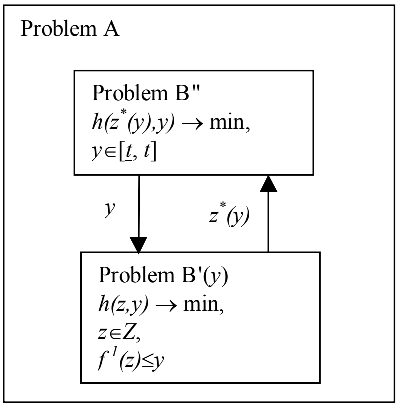

Fig. 1. Decomposition scheme.

At the lower level, for the fixed $y\in [\underline{t}, \bar{t}]$ the problems B'*(y)* of minimising function

$$h(z,y)= yf^2(z)+f^3(z)$$

on the set $\{z \in Z | f^1(z) \le y\}$ are solved.

Let *z*(y)* be a solution of problem B'*(y)*. At the upper level, the problem B" to minimise function

$$F(y)=h(z^*(y),y)$$

on $[\underline{t}, \bar{t}]$ is solved.

4.2. Methods for solving sub-problems

For solving the problem B", a special modification of "branch and bound" method (Guschinsky *et al.*, 1990) is used. In this procedure, the segment $[\underline{t},\bar{t}]$ is replaced by a set *Y* and value $y^0 \in [\underline{t},\bar{t}] \setminus Y$ such that $\min\{F(y) \mid y\in Y\cup\{y^0\}\}=\min\{F(y) \mid y\in [\underline{t},\bar{t}]\}$. The set *Y* is partitioned into intervals $Y_i = (y_i^-, y_i^+)$, $i=1,2,\ldots$.

The value $\sigma_i = F' y_i^- / y' + \beta_i (1 - y_i^- / y')$ is considered as a lower bound of the function *F(y)* on Y_i, where β_i is a lower bound of the function $f^3(z)$ on the set $\{ z \in Z \mid y_i^- < f^1(z) < y_i^+ \}$ and $F' \le h(z^*(y'), y')$ for some $y' \ge y_i^+$. As *y'*, we can accept $y \in [\underline{t},\bar{t}]$ that is the nearest point to y_i^+ and for which value *F(y)* is known.

An interval Y_{i^*} with the minimal bound σ_{i^*} is chosen. If $\sigma_{i^*} \ge F^0+\varepsilon$, then algorithm stops where ε is a required accuracy to the solution of the problem B". Otherwise, the value

$$\hat{y}_{i^*} = \min[\, y'(F(y^0) - \beta_i^3) / (F' - \beta_i^3),\ y_{i^*}^+]$$

is calculated and the set $[\hat{y}_i, y_i^+)$ is deleted from Y_{i^*}. If the obtained set Y_{i^*} is empty, then the next set with minimal bound is selected. Otherwise, a point $\tilde{y}$ in the set Y_{i^*} is chosen and $F(\tilde{y})$ is found. If $F(\tilde{y}) < F^0$, then $y^0 = \tilde{y}$. The set Y_{i^*} is replaced by two intervals $(y_{i^*}^-, \tilde{y})$ and $(\tilde{y}, y_{i^*}^+)$. Finally, for the new intervals the bound σ is calculated.

For problem B'*(y),* a procedure, which is based on a multi-criteria shortest path algorithm similar to (Dolgui *et al.,* 1999b), is used.

It is easy to see that $\underline{t} \ge \tau''+\max\{l(p)/x_2(p) \mid p\in \boldsymbol{N}\}$. To find the precise value $\underline{t}$, an algorithm, which is similar to the algorithm for the problem B", can be used.

5. A NUMERICAL EXAMPLE

The following simplified example illustrates the approach proposed. Set N of operations and their parameters are given in Table 1 ($\gamma(p)=0.7$, $\eta(p)=2$ for all operations).

Graph (N,*L*) of precedence constraints is shown in Fig. 2. It is supposed that sets $E^b=\{\{1,9\}, \{2,7\}, \{3,4\}, \{6,8\}\}$, $E^s=\{\{1,4\},\{2,5\}\}$, $H^b=\{\{3,7\}\}$, $H^s=\varnothing$, and $\tau''=0.5$, $\tau'=0.3$, $\bar{t} = 2.5$, $\alpha_l=0.2$, $m_0=3$, $n_0=3$. Graph *G* is shown in Fig. 3.

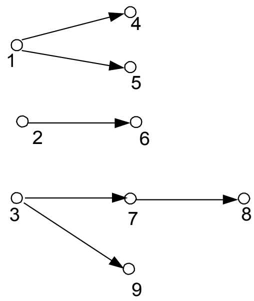

Fig. 2. Graph *(*N*,L)*.

In this graph the vertices from $\overline{V}$ are drawn by bold line. Feasible values $X(N_{kj})$ for arcs of graph G are given in Table 2. Functions $c_i(.)$, i=1,2,3,4 are given in Tables 3 and 4. The functions are defined only for such sets N_k and N_{kj} necessary to search an optimal solution in graph G .

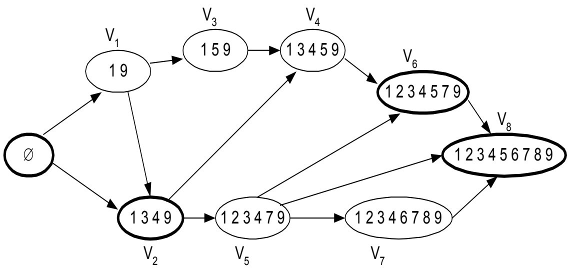

Fig. 3. Graph *G*.

Table 1 Set N of operations and their parameters

P	*l(p)*	s_1(p)	s_2(p)	δ_1(p)	δ_2(p)	s_0(p)	δ_0(p)	αi_{10}(p) + i_{20}(p)
1	100	0.175	0.218	400	550	0.2	450	0.15
2	40	0.18	0.22	250	315	0.2	270	0.08
3	128	0.25	0.33	200	305	0.3	250	0.24
4	90	0.35	0.45	170	310	0.4	250	0.15
5	21	0.05	0.07	400	500	0.06	450	0.09
6	15	0.075	0.12	200	330	0.1	250	0.06
7	30	0.08	0.12	375	500	0.1	400	0.09
8	18	0.04	0.06	375	500	0.05	430	0.09
9	115	0.2	0.3	300	400	0.25	350	0.20

Table 2 Values $X(N_{kj})$

N_{kj}	$\underline{X}(N_{kj})$	$\overline{X}(N_{kj})$
1 9	70	120
1 3 4 9	70	100
2 7	45	60
3 4	60	100
5	20	35
5 6 8	20	30
6 8	15	30

Table 3 Functions $c_i(.)$, i=1,3

N_k	$c_1(.)$	$c_3(.)$
(1,3,4,9)	0.035	0.22
(2,5,7)	0.03	0.20
(2,7),(5,6,8)	0.04	0.25
(6,8)	0.03	0.20

Table 4. Functions $c_i(.)$, i=2,4

N_{kj}	$c_2(.)$	$c_4(.)$
1,9	0.01	0.07
1,3,4,9	0.015	0.12
2,7	0.01	0.085
3,4	0.01	0.85
5	0.008	0.067
5,6,8	0.013	0.1
6,8	0.008	0.085

The following solution P^*=<(({1,3,4,9},80)), (({2,7},60), ({5,6,8},30))> of the initial problem is optimal under the condition that feasible values of $X(N_{kj})$ are divisible by 5. Then $\Theta(P^*)$=2.69 and $t(P^*)$=2.47.

6. CONCLUSION

A problem of preliminary design of transfer line has been studied. The problem is to find the number of stations and the number of spindle heads, to assign the manufacturing operations to stations and to spindle heads, and to choose the operating modes for each spindle head so that the life cycle cost per part is minimal.

The initial problem has been reduced to a parameterised path optimisation problem. The proposed approach allows to minimise the line life cycle cost under the given productivity and technological constraints.

The studied problem comes from real industrial applications. The principal parameters are included in the proposed model, but real line is characterised by a great number of parameters (for example, several work positions for each station). Further research will concern with involving complementary parameters in the model.

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