

## INFLUENCE OF HIGH PERFORMANCE VOLTAGE CONVERTERS SUPPLY UPON THE INDUCTION MOTORS BEHAVIOUR

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**Abstract:** The induction motors supply by converters leads to a distorted regime accompanied by some parasite torques and some supplementary losses. These aspects are emphasized further on because their quantitative determinations have a special influence in establishing the motors performances.

**Keywords:** Induction motors, converters, distorted regime

### 1. INTRODUCTION

When the induction motors are fed by voltage converters a distorted regime occurs, the supply voltage, respectively the supply currents being very different from a sinusoidal variation. The periodical character and a finite number of discontinuities on a period are specific to all the waveforms, so that the Fourier development can be appealed to. The superior space harmonics are not taken into account further on. In accordance with the relations (Câmpeanu, 1988), the representative phasor of the stator voltage is of the form:

$$(1) \underline{u}_s = \sum_v \underline{U}_{1v} e^{jv\omega_1 t} + \sum_{v'} \underline{U}_{1v'}^* e^{-jv'\omega_1 t}$$

where:  $\underline{U}_{1v} = -j \frac{1}{v} U_1 \sqrt{2}$ ;  $\underline{U}_{1v'}^* = j \frac{1}{v'} U_1 \sqrt{2}$ .

$$v = 2km_1 + 1 = 6k + 1 = 1, 7, 13, 19, 25, 31, \dots$$

$$v' = 2km_1 - 1 = 6k - 1 = 5, 11, 17, 23, 29, \dots$$

$$(m_1=3)$$

By applying the effects superposition principle, it is obtained:

$$(2) \underline{\Psi}_r = \sum_{v=1}^{\infty} \underline{\Psi}_{2v} e^{jv\omega_1 t} + \sum_{v'=1}^{\infty} \underline{\Psi}_{2v'}^* e^{-jv'\omega_1 t}$$

$$\underline{i}_r = \sum_{k=1}^{\infty} \underline{I}_{2k} e^{jk\omega_1 t} + \sum_{k'=1}^{\infty} \underline{I}_{2k'}^* e^{-jk'\omega_1 t}$$

where k takes values belonging to the row established for v and k' from v'.

### 2. ELECTROMAGNETIC TORQUE OF THE INDUCTION MOTOR

By introducing the values we obtain in the general expression for torque according to (Câmpeanu, 1988), it is obtained:

$$(3) M = 3p \operatorname{Re} \left[ -j \left( \sum \underline{\Psi}_{2v} e^{jv\omega_1 t} + \sum \underline{\Psi}_{2v'}^* e^{-jv'\omega_1 t} \right) \left( \sum \underline{I}_{2k}^* e^{-jk\omega_1 t} + \sum \underline{I}_{2k'} e^{jk'\omega_1 t} \right) \right] =$$

$$= 3p \operatorname{Re} \left[ -j \left( \sum \underline{\Psi}_{2v} \underline{I}_{2k}^* e^{j(v-k)\omega_1 t} + \sum \underline{\Psi}_{2v'}^* \underline{I}_{2k} e^{-j(v'-k)\omega_1 t} + \sum \underline{\Psi}_{2v} \underline{I}_{2k} e^{-j(v+k)\omega_1 t} + \sum \underline{\Psi}_{2v'}^* \underline{I}_{2k}^* e^{-j(v'+k)\omega_1 t} \right) \right]$$

or, by separating the terms in which  $v=k$  and  $v'=k'$

$$(4) M = 3p \operatorname{Re} \left[ -j \left( \sum \underline{\Psi}_{2v} \underline{I}_{2v}^* + \sum_{v=7,13,19} \underline{\Psi}_{2v} \underline{I}_{2v}^* + \sum_{v'=5,11,17} \underline{\Psi}_{2v'}^* \underline{I}_{2k'} + \sum \underline{\Psi}_{2v} \underline{I}_{2k}^* e^{j(v-k)\omega_1 t} + \sum \underline{\Psi}_{2v'}^* \underline{I}_{2k} e^{-j(v'-k)\omega_1 t} + \sum \underline{\Psi}_{2v} \underline{I}_{2k} e^{-j(v+k)\omega_1 t} + \sum \underline{\Psi}_{2v'}^* \underline{I}_{2k}^* e^{-j(v'+k)\omega_1 t} \right) \right]$$

The first term from the second member of the relation (4) is the torque  $M_1$  conditioned by the fundamental harmonics. The induction machine equations, according to (Câmpeanu, 1988) give:

$$(5) M_1 = 3p \operatorname{Re} \left[ -j \frac{R_2 I_2}{-js\omega_1} I_2^* \right] = 3p \frac{R_2 I_2^2}{s\omega_1}$$

The asynchronous torque conditioned by the time harmonic of the  $\nu$  order:

$$(6) M_\nu = 3p \operatorname{Re} \left[ -j \Psi_{2\nu} I_{2\nu}^* \right] = p \operatorname{Re} \left[ -j \frac{R_{2\nu} I_{2\nu}}{-js_\nu \omega_{1\nu}} I_{2\nu}^* \right] = 3p \frac{R_{2\nu} I_{2\nu}^2}{s_\nu \nu \omega_1}$$

For the harmonic of the  $\nu'$  order, and by doing as above it is obtained:

$$(7) M_{\nu'} = 3p \operatorname{Re} \left[ -j \Psi_{2\nu'}^* I_{2\nu'} \right] = -3p \frac{R_{2\nu'} I_{2\nu'}^2}{s_{\nu'} \nu' \omega_1}$$

By introducing in (4) the values we obtained it results:

$$(8) M = M_- + M_+$$

Where  $M_-$  is a sum of all the components from (4) affected by exponent and which condition sinusoidal oscillating torques and:

$$(9) M_- = M_1 + \sum M_\nu - \sum M_{\nu'} = M_1 + M_7 - M_5 + M_{13} - M_{11} + \dots$$

is the continuous component of the torque  $M$ , conditioned by the fundamental harmonic and the superior time harmonics of the supply voltage and the corresponding induced rotor currents.

On the basis of the relations (5), (6) it is obtained:

$$(10) \frac{M_\nu}{M_{1n}} = \frac{R_{2\nu}}{R_2} \left( \frac{I_{2\nu}}{I_{2n}} \right)^2 \frac{s_n}{s_\nu \nu} \cong k_{R2\nu} \left( \frac{I_{1\nu}}{I_{1n}} \right)^2 \frac{s_n}{\nu s_\nu}$$

From the numerical values we obtained and from the analytical expression (10), the torque decreases very much with the harmonic order  $\nu$ , although the coefficients  $k_{R2\nu}$  feel increases, so that it is possible to appreciate that  $(\sum M_\nu - \sum M_{\nu'})$  practically has a value that can be neglected in comparison with the rated torque. It is also observed that the magnitude of the torques  $M_\nu$  (in the rotation sense) and  $M_{\nu'}$  (opposite to the rotation) is practically independent on the induction machine load.

As it comes out from (8) the alternating components  $M_-$  of the torques (oscillating torques), with null average value, are also established, these torques being conditioned by the interaction between rotating fields harmonics and harmonics of other order of the rotor currents (induced by other harmonics of the rotating fields) which must taken into consideration with necessity. First of all it is necessary to follow the oscillating torques resulted from the interaction between different superior harmonics of the rotor currents and the fundamental harmonic flux. By considering  $\nu = 1$ ,  $\nu' = 0$  it is obtained:

$$(11) M_- = 3p \operatorname{Re} \left[ -j \sum \Psi_2 I_{2k}^* e^{j(1-k)\omega_1 t} + \sum \Psi_2 I_{2k'} e^{j(1+k')\omega_1 t} \right]$$

or, taking only the harmonics  $k$  and  $k'$ :

$$(12) M_- = 3p \operatorname{Re} \left[ -j \left( \Psi_{2,7} I_{2,7}^* e^{-j6\omega_1 t} + \Psi_{2,5} I_{2,5}^* e^{j6\omega_1 t} + \Psi_{2,13} I_{2,13}^* e^{-j12\omega_1 t} + \Psi_{2,11} I_{2,11}^* e^{j12\omega_1 t} \right) \right]$$

It is observed that the interaction with the rotor currents harmonics of the 5 and of the 7 order determines sinusoidal oscillating torques with the pulsation, in analogous way the harmonics of the 11, 13 order - oscillating torques with the pulsation  $12\omega_1$ , etc.

Because the currents  $I_{2\nu}$  decrease when the harmonic order increases, we are mainly interested in the first harmonics. The amplitude of the oscillating torque  $M_{-5}$  component conditioned by the product  $\Psi_2 I_{25}$  related to the rated torque is about in the ratio

$$\frac{I_{25}}{I_{2n}} \cong \frac{I_{15}}{I_{1n}} \text{ or taking into account the ratio:}$$

$$\frac{M_{-5}}{M_{1n}} = \frac{0,3I_{1n}}{5^2 \cdot 0,05I_{1n}} \cong 0,24$$

The amplitude of the component with the same pulsation  $6\omega_1$  conditioned by the harmonic  $I_{27}$  is in the ratio:

$$\frac{M_{-7}}{M_{1n}} = \frac{0,3I_{1n}}{7^2 \cdot 0,05I_{1n}} \cong 0,12.$$

The resultant amplitude of the torque of the pulsation  $6\omega_1$  is given by the sum of the two lagged sinusoidal waves (geometric sum).

It is observed immediately that the amplitudes of the torques  $M_{-11}, M_{-13}$  with the pulsation  $12\omega_1$  decrease quickly.

The other oscillating components given by  $M_{\sim} - M'_{\sim}$  have the pulsations over  $6\omega_1$  (the relations (1) and (11)) and the amplitudes much lower, because  $\Psi_{2v}$  and  $\Psi_{2v'}$  decrease at the same time with  $I_{2k}, I_{2k'}$ .

the parameters corresponding to the frequency  $f_v$  are computed by the relations:

Because the currents  $I_{2v}$  do not depend on the motor load, the oscillating torque amplitude practically does not change, irrespective of whether the motor operates without load or with rated load or whether the machine operates as generator or as motor. It is directly conditioned by the leakage coefficient,  $\sigma$ . For the considered leakage, (Câmpeanu, 1988; Măgureanu, 1985),  $\sigma = 0.05$ , we expect oscillating torques with amplitude of about  $0,25 M_N$ . At rated load the oscillating torques determine a waving of the average torque  $M = M_{-} + M_{\sim}$  of the motor. Since they do not depend on load, when it decreases, the resultant torque may change periodically its sign with unpleasant effects on the motor and the mechanical elements.

### 3. INDUCTION MOTOR SUPPLEMENTARY LOSSES

The superior time harmonics of the electromagnetic quantities produce supplementary losses into the windings and into the feromagnetic core. When computing the winding losses there are taken into account both the increase of the curenets root-mean-square, determined by the superior time harmonics, and the windings resistance increase factors for superior frequencies. The supplementary losses into the magnetic circuit impose to establish the superior harmonics of the induction into different parts of the magnetic circuit. When computing these losses for superior frequencies, the reaction of the eddy-currents into laminations has been taken into account.

#### 3.1. Induction machine parameters for superior time harmonics.

The meaning of the notations from the speciality literature is retained, being emphasized the differences that occur in the case of the parameters corresponding to the superior time harmonics of the electromagnetic quantities. The differences are determined by the strong non-uniformity of the electromagnetic field distribution into the conductor elements for the superior frequencies. This aspect is taken into account by means of the resistance increase coefficients,  $k_{Rv}$ , respectively of the reactance decrease coefficient,  $k_{Xv}$ , corresponding to the frequency  $f_v$ . By neglecting the slump phenomenon in the frontal zone of the conductors,

$$(13) R_v = k_{Rv} R = R \frac{k_{RCv} I_{Fe} + (I_{med} - I_{Fe})}{I_{med}}$$

$$(14) X_{\sigma v} = k_{Xv} v X_{\sigma} = v X_{\sigma} \frac{k_{XCv} I_{Fe} + (I_{med} - I_{Fe})}{I_{med}}$$

### 3.1.1. Stator winding parameters.

For the analyzed motors, the stator winding is made by profiled copper conductor, in two layers, with short pitch, being placed in open slots with parallel walls, the conductors being put on their width.

Having noted  $k'_{RCv}$  - the average value of the resistance increase coefficient for the case when the coil sides from the slot belong to the same phase,  $k''_{RCv}$  - the average value of the resistance increase coefficient for the case when the coil sides belong to different phases,  $q_1$  - the number of slots per pole and phase and  $v$  - the number of slots per pole and phase in which there are coil sides belonging to the same phase, it is possible to write for the whole winding according to (Cioc et al., 1994; Dordea et al., 1992; Richter, 1969):

$$(15) k_{RCv} = \frac{v K'_{RCv} + (q_1 - v) K''_{RCv}}{q_1}$$

$$(16) k_{RCv} = \varphi(\xi_v) + \frac{1}{q_1} \left[ v \frac{m^2 - 1}{3} + (q_1 - v) \frac{13m^2 - 16}{48} \right] \Psi(\xi_v)$$

For the total average coefficient of reactance decrease, some corresponding expressions are obtained analogously:

$$(17) k_{XCv} = \frac{1}{m^2} \left\{ \lambda(\xi_v) + \frac{1}{q_1} \left[ v \frac{m^2 - 1}{3} + (q_1 - v) \frac{13m^2 - 16}{16} \right] \mu(\xi_v) \right\}$$

The functions  $\varphi, \Psi, \lambda, \mu$ , of variable  $\xi_v$ , are computed with the relations known from (Câmpeanu, 1988; Cioc, et al., 1994; Dordea, et al., 1992).

### 3.1.2. Rotor winding parameters.

By taking into account (Câmpeanu, 1988; Cioc, et al., 1994; Dordea, et al., 1992; Richter, 1969), the rotor winding parameters corresponding to the superior time harmonics of the rotor currents, are established by using the expressions:

$$(18) R_{2v} = k_{R2v} R_v + \frac{R_i}{2 \sin^2 \frac{\pi p}{Z_2}}$$

$$(19) X_{2\sigma v} = v 2\pi f_1 \mu_0 l_i \sum \lambda_{2v}$$

where:  $\sum \lambda_{2v} = k_{X2v} \lambda'_{C2} + \lambda''_{C2} + \lambda_{d2} + \lambda_{f2}$ .

### 3.2 Electromagnetic quantities determined by voltage harmonics.

When an induction machine operates with a non-sinusoidal supply, the stator voltage can be decomposed in a fundamental component and a series of odd harmonics. The resultant fields into the air gap, corresponding to the superior harmonics, have low values and their influence upon the resultant field, determined by the currents fundamental, is small. The saturation degree is approximately the same for all the harmonics, so that it is possible to apply the principle of the effects superposition (Buhler, 1995; Câmpeanu, 1988; Dordea, et al., 1992; Măgureanu et al., 1985). This means that the motor behaviour can be analyzed independently for the fundamental voltage and for every harmonic term. The global answer to non-sinusoidal voltage is obtained as a sum of the individual components answers. The magnitude of the currents  $I_{1v}, I'_{2v}$ , is determined by solving the equations set corresponding to the induction machine for the voltage harmonic  $v$ .

$$(20) I_{1v} = U_{1v} \frac{Z_{1mv} + Z'_{2v}}{Z_{1v} Z_{1mv} + Z_{1v} Z'_{2v} + Z'_{2v} Z_{1mv}}$$

$$(21) I'_{2v} = -U_{1v} \frac{Z_{1mv}}{Z_{1v} Z_{1mv} + Z_{1v} Z'_{2v} + Z'_{2v} Z_{1mv}}$$

The root-mean-square value of the e.m.f. results from relation

$$(22) U_{e1v} = \sqrt{|U_{1v}|^2 - |Z_{1v} I_{1v}|^2 - 2|U_{1v}| |Z_{1v} I_{1v}| \cos \alpha_\alpha}$$

where

$$\alpha_v = \angle(U_{1v}, Z_{1v} I_{1v}) \text{ and } \alpha_v = \frac{\pi}{2} - \varphi_{1v} - \arctg \frac{R_{1v}}{X_{1\sigma v}}$$

Thus we obtain the amplitude of the harmonic  $v$  of the magnetic induction into the air gap

$$(23) B_{\delta v} = \frac{U_{e1v}}{2\sqrt{2} N_1 k_{B1} f_v t_l}$$

### 3.3. Windings supplementary losses.

The windings of the induction motors fed by converters are the place where occur some important supplementary losses, determined by two causes:

- the increase of the currents root-mean-square in comparison with the root-mean-square of the fundamental, that ensures the useful power transfer;
- the increase of the windings resistance for the currents superior harmonics.

The total losses into the stator winding are computed by the relation:

$$(24) P_{cu1} = m_1 \sum_{v=1}^{\infty} R_{1v} I_{1v}^2 = m_1 R_1 \sum_{v=1}^{\infty} k_{R1v} I_{1v}^2$$

Having noted:  $P_{cu1} = m_1 R_1 k_{R11} I_{11}^2$

$$(25) P_{cu1} = k_{pcu1} P_{cu1}$$

where (26)  $k_{pcu1} = 1 + \sum_{v=1}^{\infty} k_{R1mv} \left( \frac{I_{1v}}{I_{11}} \right)^2$

is the losses increase factor for the stator winding, and

$$(27) k_{R1mv} = \frac{k_{R1v} I_{Fe} + (l_{med} - l_{Fe})}{k_{R11} I_{Fe} + (l_{med} - l_{Fe})}$$

is the resistance increase factor corresponding to the harmonic  $v$  related to the value corresponding to the fundamental.

The total losses into the rotor winding are computed analogously:

$$(28) P_{cu2} = m_2 \sum_{v=1}^{\infty} R_{2v} I_{2v}^2 = m_2 R_2 \sum_{v=1}^{\infty} k_{R2v} I_{2v}^2$$

Having noted  $P_{cu21} = m_2 R_2 k_{R21} I_{21}^2$

$$(29) P_{cu2} = k_{pcu2} P_{cu21}$$

where (30)  $k_{pcu2} = 1 + \sum_{v=1}^{\infty} k_{R2mv} \left( \frac{I_{2v}}{I_{21}} \right)^2$

is the losses increase factor for the rotor winding, and

$$(31) k_{R2mv} = \frac{k_{R2v}}{k_{R21}}$$

is the resistance increase factor corresponding to the harmonic having the frequency  $s_v f_1$  related to the value corresponding to the fundamental.

The windings supplementary losses are established by the relation:

$$(32) P_{cus} = P_{cu1s} + P_{cu2s} = (k_{pcu} - 1) P_{cu}$$

with:  $k_{pcu}$  - the losses increase factor for the machine windings.

### 3.4. Ferromagnetic core supplementary losses.

The superior time harmonics of the supply voltage determine rotating magnetic fields having the angular

speed  $v\Omega_1$  and respectively  $-v'\Omega_1$  relatively to the stator and the angular speed  $(v-1)\Omega_1$  and respectively  $-(v-1)\Omega_1$  relatively to the rotor, producing supplementary losses into the stator yoke and teeth, respectively into the rotor teeth and pulsation supplementary losses, respectively surface supplementary losses. It must be noted that every pair of stator harmonics with the order  $v = 2km_1 \pm 1$  produces magnetic fields rotating with the same speed relatively to the rotor. For the iron supplementary losses calculus the following hypotheses are considered:

- the air gap is constant;
- there is considered the magnetic permeability corresponding to the time and space fundamentals.

#### 3.4.1. Superior harmonics of the magnetic inductions.

Having known the magnetic inductions into every part of the magnetic circuit, the iron losses can be computed. For the stator ferromagnetic core the inductions maximum values in time and those average in space can be computed by the relations we know (Câmpeanu, 1988; Cioc, et al., 1994; Dordea, et al., 1992).

- into the air gap,  $B_{\delta v}$  ;
  - into the stator yoke,  $B_{j1v}$  ;
  - into different sections "x" of the teeth,  $B'_{d1xv}$  .
- The rotor core inductions are computed by the relations from (Dordea, et al., 1992):
- into the rotor yoke,  $B_{j2v}$  ;
  - into different sections "x" of the teeth,  $B'_{d2xv}$  .

#### 3.4.2. Supplementary losses.

When computing the iron losses for high frequencies, it is necessary to take into account the eddy-currents reaction, that leads to the fact that the induction has not an uniform distribution on the lamination thickens  $\Delta$ , decreasing from the lateral walls to the median plane.

a) Since the inductions of harmonic  $v$  have a sinusoidal time variation, the supplementary losses into different parts of the stator magnetic circuit, determined by these harmonics, are computed by the relations presented in (Dordea, et al., 1992; Richter, 1969):

$$(33) P_{Feprv} = P_{j1v} + P_{d1v}$$

$$(34) P_{j1v} = (k_{Hj1} \cdot P_{Hj1v} + k_{Fj1} \cdot P_{Fj1v}) \cdot G_{j1}$$

$$(35) P_{d1v} = (k_{Hd1} \cdot P_{Hd1v} + k_{Fd1} \cdot P_{Fd1v}) \cdot G_{d1}$$

In these relations  $G_{j1}$  is the weight of the stator yoke magnetic core,  $G_{d1}$  is the weight of the stator teeth,  $k_{Hj1}$ ,  $k_{Fj1}$ ,  $k_{Hd1}$ ,  $k_{Fd1}$ , are the coefficients taking into account the processing degree of the magnetic core and the conditions in which the hysteresis losses, respectively the eddy-currents losses occur, shown in (Dordea, et al., 1992; Richter, 1969) and:

$$(36 \text{ a, b}) \quad P_{Hj1v} = \frac{vf_1}{100} (\alpha B_{j1v} + \beta B_{j1v}^2)$$

$$P_{Hd1v} = \frac{vf_1}{100} (\alpha B'_{d1medv} + \beta B'_{d1medv})$$

and

$$(37., \text{ a, b}) \quad P_{Fj1v} = \sigma_F \cdot v^2 \cdot B_{j1v}^2 \cdot k_{mv}$$

$$P_{Fd1v} = \sigma_F \cdot v^2 \cdot B'_{d1medv} \cdot k_{mv}$$

are the hysteresis losses, respectively the eddy-currents losses per weight unit.

Having noted with  $P_{Fepr1}$  - the stator ferromagnetic core losses determined by the fundamental time harmonic of the electromagnetic quantities and with:

$$(38) \quad P_{Feprv} = \sum_{v>1} P_{Feprv}$$

the supplementary losses determined by the superior time harmonics in the same part of the magnetic circuit, the total main losses into the stator ferromagnetic core can be established with the relation:

$$(39) \quad P_{Fepr} = P_{Fepr1} + P_{Feprv} = k_{pFepr} \cdot P_{Fepr1}$$

b) At the considered induction motors, the stator superior time harmonics, with the frequency  $f_{1v}$ , respectively  $f_{1v'}$ , produce rotor superior time harmonics with the frequency:

$$(40) \quad f_{2v} = s_v f_{1v}$$

$$(41) \quad f_{2v'} = s_{v'} f_{1v'}$$

The frequencies  $f_{2v}$  and  $f_{2v'}$  have high values and the losses into the rotor ferromagnetic core may not be neglected anymore. The losses into the rotor magnetic core, determined by the superior harmonics having the frequencies we mentioned, occur into parts similar with those of the stator magnetic circuit (yoke, teeth), being computed by relations similar with the relations (33) ÷ (37), in which are involved the inductions, the frequencies (the frequencies order) and the iron weights corresponding to the parts of the magnetic circuit we considered. The supplementary iron losses produced into the rotor

ferromagnetic core by the superior time harmonics are:

$$(42) \quad P_{Fers} = \sum_{v=1}^{\infty} P_{Ferv} = \sum_{v=1}^{\infty} (P_{Fejev} + P_{Fed2v})$$

c) At the induction motors, both for no-load operation and for the load operation, there occur supplementary surface iron losses because of the magnetic induction oscillations. They are produced on the surfaces from the air gap of the ferromagnetic cores, being caused by the air gap induction pulsation produced by the fact that there are slots on the two armatures. The value of these losses depend on the amplitude of these pulsations, therefore on the slots opening and on their frequency given by the product  $zn_l$ . The fact that there are slots on both armatures leads to some supplementary pulsation losses into the teeth. These losses are produced into the mass of the stator teeth because of the rotor slots and respectively into the mass of the rotor teeth owing to the stator slots. Moreover, the superior time harmonics  $B_{\delta v}$ ,  $B'_{d1medv}$ ,  $B'_{d2medv}$  lead to some supplementary surface and pulsation losses into the teeth. These losses value is established by using the relations recommended for the fundamental (Cioc, et al., 1994; Dordea, et al., 1992; Richter, 1969) with the values of the corresponding inductions. Having noted with  $P_{Fes01}$  - the supplementary surface and pulsation iron losses into the teeth for no-load operation, produced by the fundamental time harmonic of the electromagnetic quantities and with:

$$(43) \quad P_{Fes0s} = \sum_{v>1}^{\infty} P_{Fes0v}$$

the supplementary losses determined by the superior time harmonics, the total supplementary losses can be established by the relation:

$$(44) \quad P_{Fes0s} = k_{pFes0} \cdot P_{Fes01}$$

We have noted with:

$$(45) \quad k_{pFes0} = 1 + \frac{P_{Fes0s}}{P_{Fes01}}$$

the supplementary surface and pulsation losses increase factor.

The total iron losses in the case of the induction motors fed by static converters, produced by the time harmonic of the electromagnetic quantities will be:

$$(46) \quad P_{Fev} = P_{Feprv} + P_{Ferv} + P_{Fes0v}$$

Analogously, having noted  $p_{Fe1}$  the total iron losses produced by the fundamental time harmonic, the iron losses increase factor can be determined:

$$(47) k_{pFe} = 1 + \frac{\sum_{v>1}^{\infty} P_{Fev}}{P_{Fe1}}$$

the total iron losses in this case being:

$$(48) P_{Fe} = k_{pFe} \cdot P_{Fe1}$$

with  $k_{pFe}$ - the total iron losses increase factor.

#### 4. RESULTS AND CONCLUSIONS.

By using the MATLAB programs carried out for modelling of the voltage inverter with two levels with pre-calculated modulation (**Impea**), **cmpeauf.m**, of the voltage inverter with three levels and floating neutral with annulling of the 5-th harmonic (**Itnt1**), **ctnnf1uf.m**, of the inverter with three levels and floating neutral with annulling of the 7-th harmonic (**Itnnf2**), **ctnnf2uf.m** and of the voltage inverter with three levels and harmonics annulling (**Itnea**), **ctneauf.m**, there have been obtained the waveforms of the phase voltage and their harmonics spectrum for different types of the induction motor commands and for different frequencies. The induction motor we considered has the following rated data:  $P_N = 155 \text{ kW}$ ,  $U_{1N} = 500 \text{ V}$ ,  $n_1 = 1500 \text{ r.p.m}$ ,  $I_{1N} = 233,46 \text{ A}$ .

The table 1 gives the total distortion factors obtained for the phase voltage waveforms.

Table 1

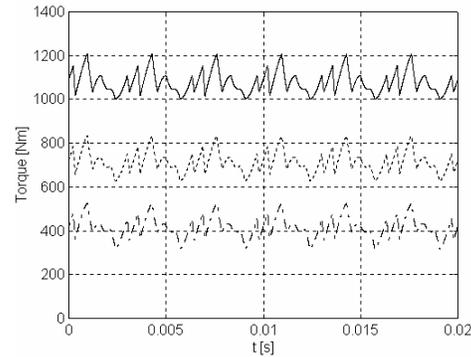
Inverter Type	Command type	Frequency	Total distortion factor
Impea	$U_{1N}$	$f_1 = 50\text{Hz}$	0,4029
Impea	$U_1 / f_1 = ct$	$f_1 = 5\text{Hz}$	0,4155
Impea	$\Psi_{1h} = ct$	$f_1 = 5\text{Hz}$	0,4568
Itnnf1	$U_{1N}$	$f_1 = 50\text{Hz}$	0,1665
Itnnf2	$U_{1N}$	$f_1 = 50\text{Hz}$	0,1652
Itnea	$U_{1N}$	$f_1 = 50\text{Hz}$	0,1441

It is observed that the voltages provided by the inverters with three levels are less distorted than those provided by the inverter with two levels with pre-calculated modulation and at the same time for low frequencies, owing to the overmodulation, the total distortion factors are higher.

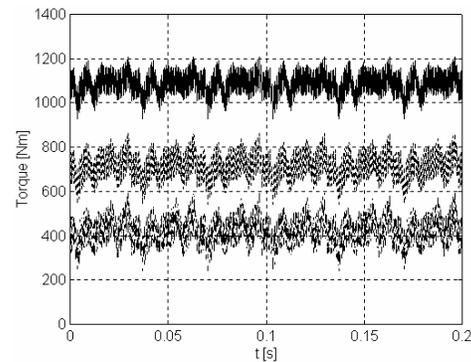
From the analysis of the values  $U_{1v} / U_{11}$  it comes out that their magnitude depends on the inverter type and on the frequency value. It comes out that the superior harmonics values are much lower at the

inverter with three levels and harmonics elimination (Itnea), than at the other types of inverters.

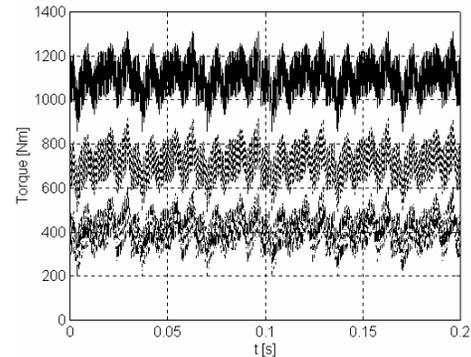
The influence of certain superior harmonics of the voltage and the magnitude of their amplitude, corresponding to every type of inverter, also occurs in the torque variation form, represented in the figure 1.a ÷ f, leading to the oscillating torques of which amplitude related to the rated torque value ( $M_N$ ) is given in the table 2. The torques developed by the motor for three values of the resistant torque  $M_r = M_N$  (----),  $I_{s1(a)} = 233,46 \text{ A}$ ;  $M_r = 0,6M_N$  (····),  $I_{s1(b)} = 169,65 \text{ A}$ ;  $M_r = 0,3M_N$  (·-·-·-),  $I_{s1(c)} = 126,08 \text{ A}$ ; are shown in the figure 1.a-f.



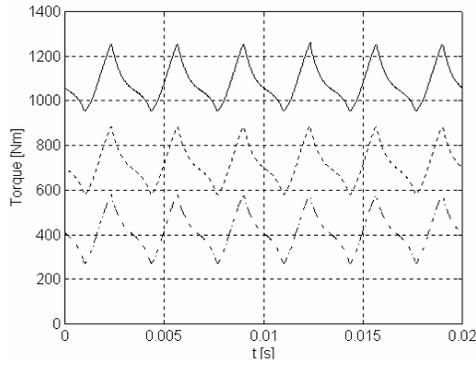
a) Impea,  $U_1 = U_{1N}$ ;  $f_1 = 50\text{Hz}$



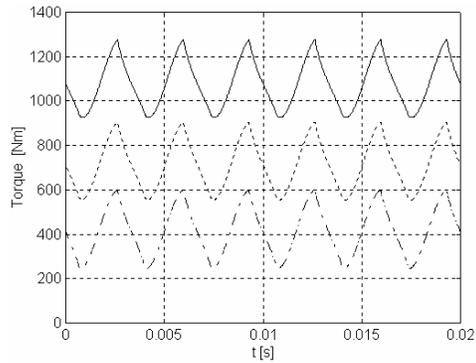
b) Impea,  $U_1 / f_1 = ct$ ;  $f_1 = 5\text{Hz}$



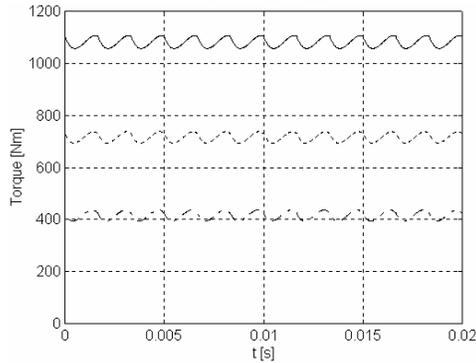
c) Impea,  $\Psi_{1h} = ct$ ;  $f_1 = 5\text{Hz}$



d) Itnnf1,  $U_1 = U_{1N}$ ;  $f_1 = 50\text{Hz}$



e) Itnnf2,  $U_1 = U_{1N}$ ;  $f_1 = 50\text{Hz}$



f) Itnea,  $U_1 = U_{1N}$ ;  $f_1 = 50\text{Hz}$

Table 2 Oscillating torques amplitudes for the load  $I_{s1(a)}$

Inverter Type	Command	Frequency	$\Delta M_{\sim\text{max}}$ $i_{s1(a)}$
Ipmea	$U_{1N}$	$f_1=50\text{Hz}$	0,084967 $M_N$
Ipmea	$U_1/f_1=ct$	$f_1=5\text{Hz}$	0,149253 $M_N$
Ipmea	$\Psi_{1h} = ct$	$f_1=5\text{Hz}$	0,188405 $M_N$
Itnnf1	$U_{1N}$	$f_1=50\text{Hz}$	0,138888 $M_N$
Itnnf2	$U_{1N}$	$f_1=50\text{Hz}$	0,166666 $M_N$
Itnea	$U_{1N}$	$f_1=50\text{Hz}$	0,025974 $M_N$

It is remarked that the oscillating torques amplitudes are placed under the values estimated in § 2, being dependent on the values of the electromagnetic quantities superior harmonics resulted as a

consequence of induction motor supply from every type of inverter. It comes out again the performances of the inverter with three levels with harmonics elimination (Itnea), the oscillating torques amplitude that results being the lowest one.

It is noticed that the voltages provided by the inverters with three levels are less distorted than those provided by the inverter with two levels with pre-calculated modulation. At the same time, for low frequencies owing to the over-modulation, the total distortion factors are increased. By using the program *cpscpmi.m*, achieved on the basis of the relations from § 3, there have been determined the supplementary losses in the windings and in the magnetic circuit of the motor that occur when supplying it from the considered inverters, for two values of the stator current fundamental  $I_{s1(a)} = 233,46\text{A}$  and  $I_{s1(c)} = 126,08\text{A}$ . The results are presented in the tables 3 and 4, where  $k_{pcu1}$ ,  $k_{pcu2}$ ,  $k_{pcu}$ ,  $k_{pFepr}$ ,  $k_{pFes0}$ ,  $k_{pFe}$ ,  $k_p$  are the losses increase factors, according to the relations from § 3.

Table 3 Losses increase coefficients for the load  $I_{s1(a)}$

Increase coefficient	Impea $f_1 = 50\text{Hz}$ $U_{1N}$	Impea $f_1 = 5\text{Hz}$ $U_1/f_1 = ct.$	Impea $f_1 = 5\text{Hz}$ $\Psi_{1h} = ct$
$K_{pcu1}$	1,1417	1,0236	1,0514
$K_{pcu2}$	1,1708	1,3489	1,4103
$k_{pcu}$	1,1503	1,0322	1,0668
$k_{pFepr}$	1,0102	1,0056	1,0068
$k_{pFes0}$	1,0010	1,0001	1,0002
$k_{pFe}$	1,0067	1,0043	1,0060
$K_p$	1,0829	1,0313	1,0642
Increase coefficient	Itnnf1 $f_1 = 50\text{Hz}$ $U_{1N}$	Itnnf2 $f_1 = 50\text{Hz}$ $U_{1N}$	Itnea $f_1 = 50\text{Hz}$ $U_{1N}$
$K_{pcu1}$	1,0369	1,0472	1,0217
$K_{pcu2}$	1,0926	1,1360	1,0402
$k_{pcu}$	1,0534	1,0735	1,0298
$k_{pFepr}$	1,0018	1,0020	1,0013
$k_{pFes0}$	1,0001	1,0001	1,0001
$k_{pFe}$	1,0013	1,0014	1,0009
$K_p$	1,0290	1,0399	1,0163

For all the inverters we considered it is noticed that irrespective of the load value, the increase of the supplementary iron losses is less than the values estimated in the literature owing to the low values of the amplitudes of the harmonics 5 – 17 from the stator form.

For all the considered inverters there is remarked the increase of all the losses increase coefficients at the same time with the load decrease, owing to the electromagnetic quantities determination

Table 4 Losses increase coefficients for the load I<sub>st(c)</sub>

Increase coefficient	Impea $f_1 = 50\text{Hz}$ $U_{IN}$	Impea $f_1 = 5\text{Hz}$ $U_1/f_1 = ct.$	Impea $f_1 = 5\text{Hz}$ $\Psi_{1h} = ct$
$k_{pcu1}$	1,4321	1,0739	1,1332
$k_{pcu2}$	2,0029	2,1509	2,2473
$K_{pcu}$	1,5344	1,1011	1,1735
$k_{pEpr}$	1,0398	1,2421	1,0068
$k_{pEs0}$	1,0306	1,2366	1,0002
$k_{pFe}$	1,0296	1,1397	1,0060
$K_p$	1,1318	1,1041	1,1534
Increase coefficient	Itnnf1 $f_1 = 50\text{Hz}$ $U_{IN}$	Itnnf2 $f_1 = 50\text{Hz}$ $U_{IN}$	Itnea $f_1 = 50\text{Hz}$ $U_{IN}$
$K_{pcu1}$	1,1158	1,1483	1,0681
$K_{pcu2}$	1,5855	1,8602	1,3107
$k_{pcu}$	1,1969	1,2713	1,1100
$k_{pEpr}$	1,0314	1,0315	1,0309
$k_{pEs0}$	1,0297	1,0297	1,0296
$k_{pFe}$	1,0241	1,0243	1,0237
$K_p$	1,0607	1,0785	1,0398

From the analysis of the tables 3 and 4 it comes out that the smallest values for the losses increase coefficients have been obtained when the induction motor is supplied from the three-level inverter with harmonics annulling (Itnea).

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