

# NONLINEAR APPROACHES REGARDING MECHANICAL AND THERMAL BEHAVIOUR OF VISCO-ELASTIC VIBRATION ISOLATORS

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## ABSTRACT

*Thermal effects inside the vibration elastomeric isolators have usually been neglected and this aspect can lead to certain differences between simulations and experiments. Continuous damping during the working cycle leads to an internal transfer of energy from mechanical movement to virtually infinitesimal punctual heating sources distributed into the entire isolator volume. This continuous tuning between the dynamics and the damping component can lead both to a stable and to an unstable behaviour in respect with the ratio between the external excitation frequency and the system natural frequency. These phenomena can be dignified through an additional terms added to the governing equations of the vibration isolation system. This paper deals with passive vibration isolation area and presents a set of nonlinear approaches regarding the mechanical and thermal phenomenon inside the elastomeric isolators during the exploitation time. The main idea consists in the continuous functional correlations between thermal and mechanical phenomena, and their influences upon the global dynamic characteristics of vibration isolators. Parametrical expressions were assumed from real experiments, directly or by means of computational techniques. Both lumped parameters model and finite element method simulations were performed in order to validate practical models.*

**KEYWORDS:** vibration isolation, nonlinear mechanics, rheology, thermal effects

## 1. INTRODUCTION

Thermal effects inside the vibration elastomeric isolators have usually been neglected. This aspect can lead to certain differences between simulations and experiments. Assessment of typical characteristics of elastomeric materials assures an approximate evolution due to external dynamic loads. Linear dependences of elastic and viscous components with displacement and velocity respectively, put into evidence the global dynamics of the system, but cannot estimate the isolator intrinsic behaviour

regarding particular changes in material characteristics. Continuous damping during the working cycle lead to an internal transfer of energy from mechanical movement to virtually infinitesimal punctual heating sources distributed into the entire isolator volume. A part of this damped energy will have been transferred through the material to isolator surroundings, while the other part assists the material characteristics changes. Obviously, the base material behaviour under the dynamic loads will acquire a certain shape according to the instantaneous characteristic parameters values. This continuous tuning between the

dynamics and the damping component can lead both to a stable and to an unstable behaviour in respect with the external excitation pulsation respecting the system natural pulsation. These phenomena can be dignified through an additional terms added to the governing equations of the vibration isolation system.

Based on the experimental tests developed by the author using rubber-based elements with different structural configurations, it was framed the idea according to which the previous factors have a great impact on the global behaviour of the vibration isolation devices. Continuous shifting of dynamic characteristic implies different outputs corresponding to the same input in respect with the time parameter. In addition, the behaviour changes have been affected by a certain random character, which comes to complicate the entire evaluation of the isolation performances for the *base-structure* ↔ *isolator* ↔ *super-structure* system.

In the frame of these remarks, it was compiled a set of nonlinear models which had been able to simulate an appropriate behaviour of vibration isolation systems comparative to experimental results. The first approaches suppose only the nonlinear characteristics of the basic materials. Computational analysis and instrumental validations were performing based on these hypotheses.

Following the previous results and taking into account the latest instrumental tests developed on experimental laboratory setups it was added the thermal component into the computational assessments. Hereby, the nonlinear differential equations acquire additional terms that can modify continuously the instant characteristics of basic materials with respect to conservative and/or dissipative components.

Depending on the elastomer-type composites used for the passive vibration isolation element, one or both of the stiffness and damping components can get thermal influences so that an overall simulation, evaluation and analysis become a *complex multiple inputs - multiple outputs* (C-MIMO) computational system.

In this paper it will be presented a group of thermo-mechanical assessments regarding the dynamics of passive vibration rubber-based isolators. Different structural configurations were adopted and different external dynamic loads were supposed to simulate external disturbing actions. Computational techniques were combined with instrumental analysis for approaching and tuning the parametrical terms of constitutive equations.

## 2. FUNDAMENTALS RELATED TO THERMO-MECHANICAL INFLUENCES

The first part of this analysis was developed based on a single degree of freedom (SDoF) dynamic system schematization. The study supposes the reference SDoF system given by a temperature-independent internal resistant force. The second SDoF system has both dissipative and conservative temperature-dependent terms. It had to be mentioned that damping with respect to Rayleigh-type law leads to a simplified analysis so that the author uses a generally loss factor-type damping.

The differential equation of this system in terms of independent variable named  $\delta$ , with both dissipative and conservative components, respects the follow formulation

$$m\ddot{\delta} + c_1\dot{\delta} + c_2\delta = Q_{ext}(t), \quad (1)$$

where  $m$  denotes the system mass,  $c_i$  denotes the damping parameter (for  $i=1$ ), respectively the stiffness parameter (for  $i=2$ ), and  $Q_{ext}$  represents the external dynamic perturbation. The reference system acquired  $c_{io}$  values for both elastic and dissipative parameters.

The temperature-dependent system respects the following law for  $c_i$  parameters

$$\frac{c_i}{c_{io}} = 1 + \xi_i e^{\beta_i(\theta_o - \theta)}, \quad i=1..2, \quad (2)$$

where index  $i$  respects the previous convention and parameters  $\xi_i$  and  $\beta_i$  denote the temperature influence coefficients. Null values for both coefficients imply the reference system behaviour with  $c_i = c_{io}$ .

The analysis of the SDoF system evolution based on Eqn. (1) was performed with output parameter as absolute differences between instantaneous displacements of reference and temperature-dependent systems. This analysis was following up the imminence of resonance phenomenon taking into account the shifting of natural frequency and the continuous changes of damping of the basic SDoF system due to temperature influences. Both steady and unsteady states were evaluated. The ratio between excitation and natural frequencies denoted by  $R$  was supposed to be an appropriate parameter for monitoring the evolution of relative displacement.

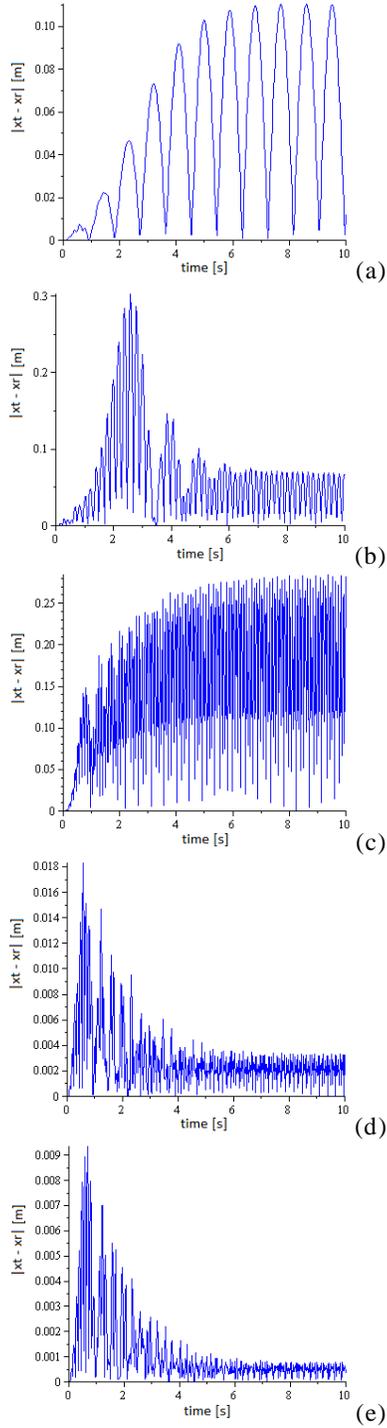


Fig. 1. The instantaneous absolute differences between evolution of theoretic and realistic displacements during the first ten seconds of transitory state for an SDoF dynamic system. The cases have the following signification (a)  $r = 10\%$ ; (b)  $r = 50\%$ ; (c)  $r = 100\%$ ; (d)  $r = 150\%$ ; (e)  $r = 200\%$ .

Diagrams in Fig. 1 depict five discrete cases with respect in  $R$  parameter, including the

resonance case (see Fig.1.c) with  $R = 100\%$ . According to Fig. 1 legend cases (a) and (b) denote pre-resonance status. In addition, cases (d) and (e) denote post-resonance status.

Comparative analysis of diagrams in Fig. 1 show that apart from (c) case, the others contain similar qualitative evolution, which means that resonance has passed through for the first 2...3 seconds from the process started.

A quantitative analysis of these diagrams reveals that even the resonance-passing period has roughly the same value, the relative displacement magnitudes have acquired greatly dangerous values for pre-resonance cases. The passing period is relatively short but the maximum values of the relative displacement exceed very much the bearing capacity of these elements so that it is impossible to exceed the resonance period while maintaining structural and functional integrity. In the same time, for the post-resonance cases, small values for relative displacement were acquired. Hereby the resonance-passing period denotes only a regular transitory state with minimal influences on the dynamic system in terms of structural integrity and functional conformity.

### 3. COMPUTATIONAL ANALYSIS

The second part of the analysis supposes a thermo-mechanical behaviour analysis of a rubber-based isolator during the regular exploitation time. The goals of this research consist of identifying heating sources inside the element due to damping and evaluating internal temperature changing.

An appropriate approach has to includes heating equation for internal dissipation inside the isolator

$$\rho c \frac{\partial \theta}{\partial t} + \nabla(-\lambda \nabla \theta) = \psi, \quad (3)$$

and convection on free boundaries including the loaded and constrained areas of the element

$$-\lambda \nabla \theta = \alpha (\theta - \theta_{ext}). \quad (2)$$

In Eqns. (3) and (4), the term  $\rho$  denotes material density,  $c$  is the heat capacity,  $\theta$  denotes the temperature inside the element,  $\theta_{ext}$  is the ambient temperature,  $\lambda$  is the heat conductivity vector containing the values for (x,y,z) axis,  $\alpha$  denotes the natural convection vector also containing the values for (x,y,z) axis.

The function  $\psi$  in Eqn. (3) denotes the internal heating sources inside the elastomeric element due to the damping phenomenon. This

volumetric dissipated power produced by the internal damping changes the internal temperature field inside the material. A continuous evaluation of the damping parameter during the analysis can be made using 2D or 3D meshing according to the initial hypothesis.

For example, the diagram in Fig 2 shows the strain energy density (see color map in Fig.2) and the heat flux (see arrows in Fig.2) for sectional orthogonal planes within a parabolic cylinder shape isolator. For simulation it was used a simple configuration of two cones trunks with the lowest common basis.

In the next part will be presented a comparative study of three types of synthetic isoprene rubber-based elements. The main hypothesis supposes a constant volume of material and various shapes such as rectangular prism, regular cylinder and parabolic cylinder. Basic element had a cubical shape with 0.05 m edge length. The other elements have maintained constant only the height dimension at 0.05 m. The heating due to damping and the internal temperature field were the pair parameters evaluated for each case.

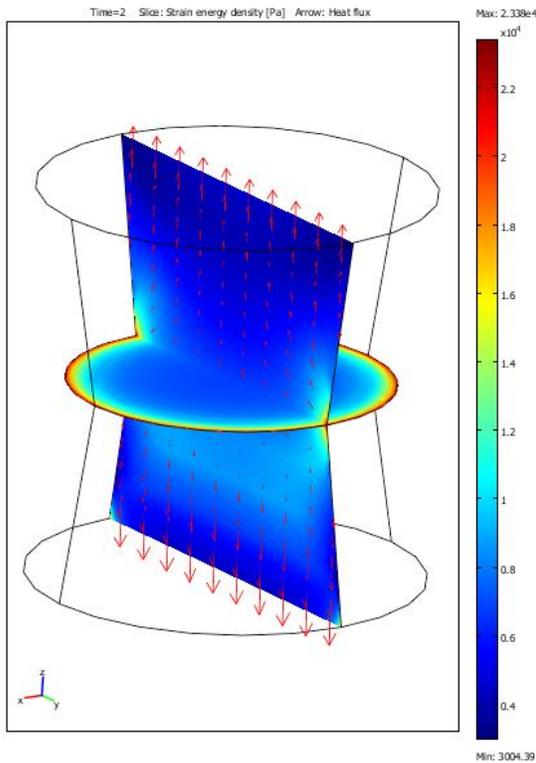


Fig. 2. Distribution of strain energy density (color map) and heat flux (arrows field) within the orthogonal sections of a profiled cylinder isolator

Supposing the longitudinal section within the isolator in Fig. 3 has been depicted the heat generation due to damping for parabolic

cylinder (see Fig.3.a) and regular cylinder (see Fig.3.b). In addition, the evolution of the same parameter along the longitudinal axis of each elastomeric element has been depicted in Fig. 4.

It has to be mentioned that the entire analysis was performed for a short time period of 2 seconds with the step of 0.05 s.

The comparative analysis of diagrams in Fig. 3 and Fig. 4 denotes an unusual evolution for the case corresponding to the profiled cylinder. For regular shapes of prism and cylinder result roughly similar evolutions of heat generation due to damping. The profiled cylinder situation provides different qualitative and quantitative evolutions comparative with the others. Hereby the heating internal sources have a minimum value inside the element and a maximum values on the boundaries.

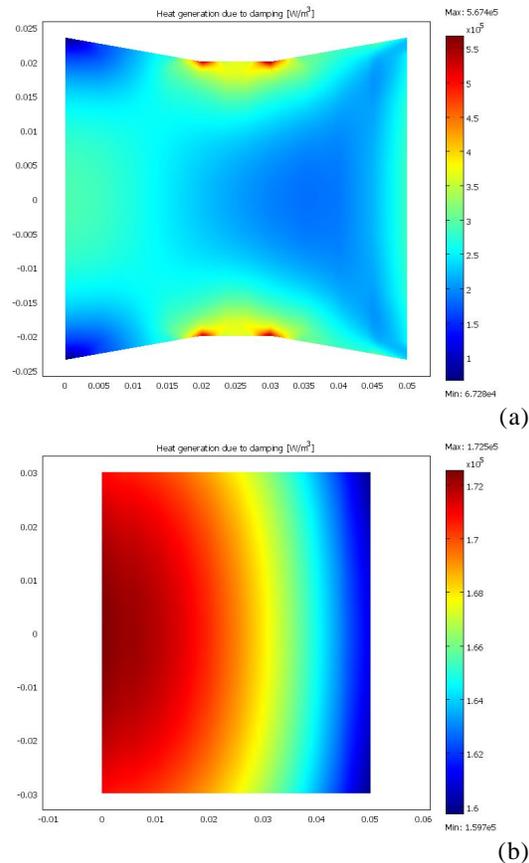


Fig. 3. Distribution of the heat generation due to damping within the longitudinal section through a profiled cylinder (a) and a regular cylinder (b)

The simulation results can be explained by the existence of convergence points on the interface boundaries for the profiled cylinder and the lack of these points for the regular shapes. In addition, the graphs in Fig. 4 reveal that the maximum values of heating due to

damping for regular shapes are lower than the minimum value of the same parameter for profiled cylinder, and this fact sustains the previous explanation. The temperature fields and the heat flux inside the proposed three elements have been depicted in Fig. 5. There have been assumed three vertical sections equally and symmetrically distributed.

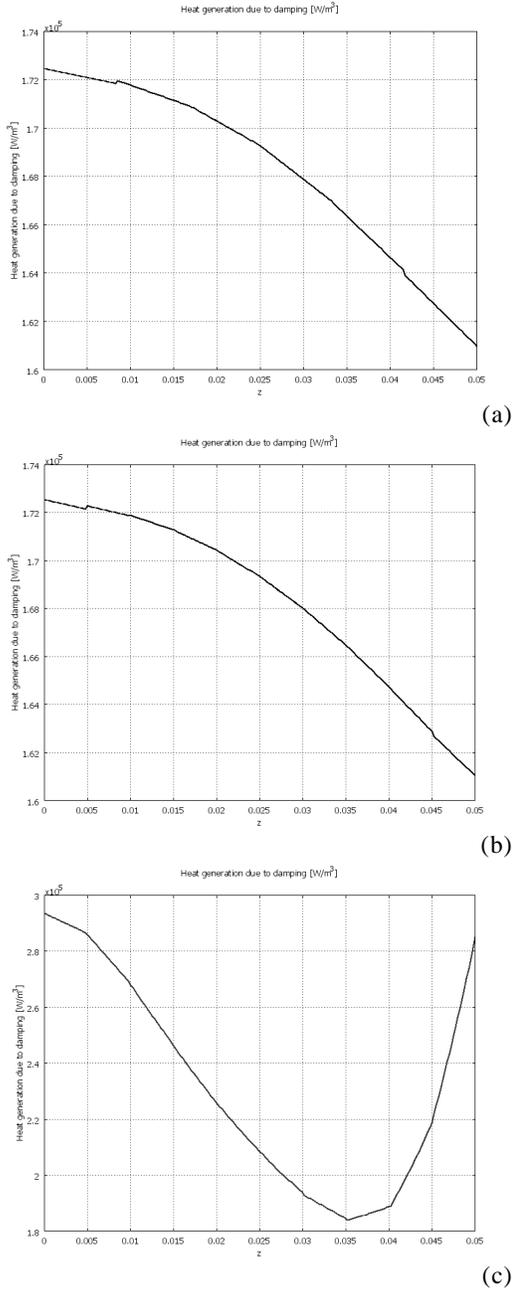


Fig. 4. Steady state evolutions of the heating due to damping parameter along the longitudinal axis of the elastomeric element. The cases have the following signification (a) *rectangular prism*; (b) *regular cylinder*; (c) *parabolic cylinder*.

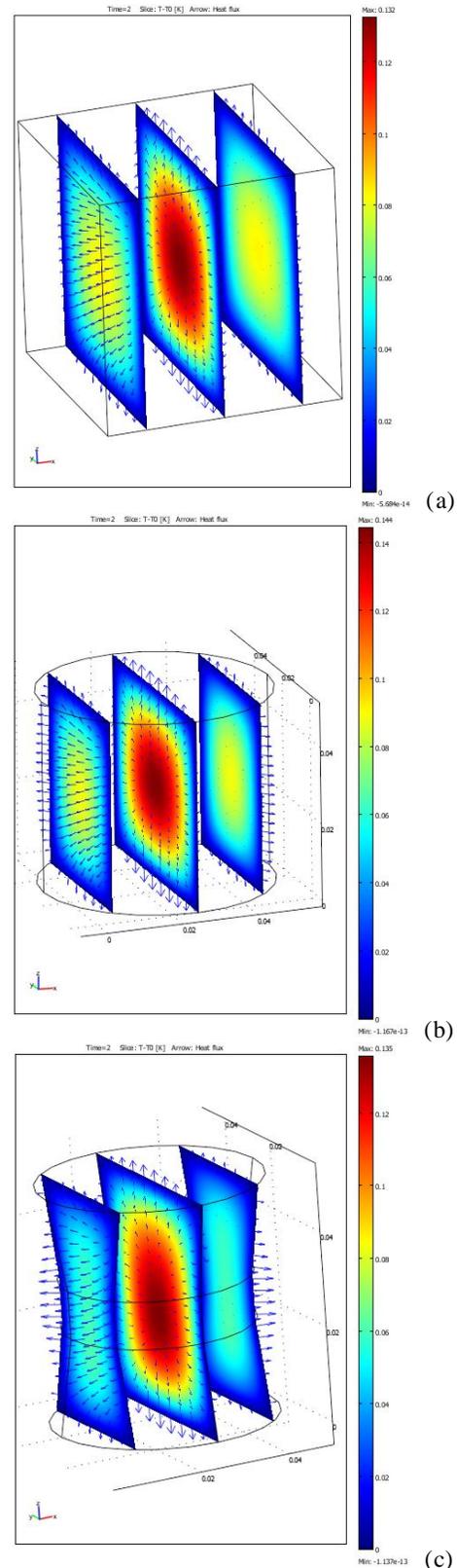


Fig. 5. Inside temperature (see color maps) and heat flux (see arrows fields) distributions for a rectangular prism (a), regular cylinder (b) and parabolic cylinder (c) shapes.

The comparative analysis of diagrams in Fig. 5 shows an identical behaviour for each case, with some differences concerning the extreme values for the monitored parameters. The maximum value of temperature changes for the cases presented in Fig. 5 reveals that regular cylinder shape induces higher modifications than the others do. However, taking into account that the differences have been less than 10% and the parabolic cylinder shape provides some other functional advantages it results that this last shape type is able to assure a suitable functional characteristic for the isolation system.

#### 4. CONCLUSIONS

Regarding the observations performed in the previous paragraphs, the first concluding remark frames the fact that regular shapes provide a uniform distribution of temperature gradient inside the material while the irregular shapes have convergence points both inside, and on the external boundaries of the element. The strain variation acquires various characteristics and implies variable damping along the orthogonal axis so that it results an inhomogeneous field of heating sources distribution inside the irregular shape elements.

According to experimental measurements related to the working temperature of viscous and elastic vibration isolators and the continuous changing of the dynamic characteristics for insulation systems including these elements, the final concluding remark dignifies both the necessity and the opportunity of the presented analysis. The comparative analysis between computational results and realistic behaviour reveals a good correlation between the models and practice. Regarding the influences of the dissipative component on the entire dynamic evolution of the elastomeric-based elements, it has to be highlighted the great importance of cold state, respectively hot state natural frequencies for the practical evaluations, integrated with both theoretical and experimental basics and validations.

Future developments of this research must supply a comprehensive nonlinear law for conservative and dissipative characteristics

with thermal influences. In addition, the influences of structural shapes on the heating generated by damping phenomenon will be further studied in order to obtain a set of functional laws with large serviceable application.

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