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THE ANALYSIS OF THE CAM MECHANISMS USING MATRIX CALCULATIONS

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ABSTRACT

The present study presents the kinematic analysis of a mechanical structure created by cam mechanism RR-CC. It was determined the position, velocity and angular acceleration of the tapped, as depending on the position and angular acceleration of the cam. The method consists in setting up matrix closing relations to determine the positions of the cam which, through subsequent derivatives, can generate a distribution of velocities and accelerations.

KEYWORDS: cam mechanisms, replacing mechanisms, cam lever

1. GENERAL CONSIDERATIONS

Kinematics analyses of mechanisms deal with the determination of many parameters like position, speeds and accelerations of mechanism elements (e.g. position angles, speed and angular acceleration in the case of elements with rotation and movements, speed and linear acceleration in the case of elements with translation).

These parameters are depending on the angle of position of leading elements of his rotation or linear movement in the case the leading element executes the translational motion.

When elements execute rotational or plane parallel motion, there can be calculated the trajectories, speed and linear accelerations of different characteristic points of these elements function of the leading elements position.

For example, the kinematic analysis of a cam mechanism assumes that we know the dimensions of the kinematic elements and the moving pattern of the driving element, and ware to determine the moving pattern of the driven element (cam lever).

Two methods are available for the kinematic analysis of a cam mechanism [9]:

- direct method, applicable for simple cases, when the upper coupling results from the curves contact: circle, line, point;
- the method of replacing mechanisms (the method of kinematic congruence), useful for mechanisms containing bars and cams for which the formulas for curvature radius characterizing the profiles in contact are known.

2. DIRECT METHOD OF ANALYSIS

The direct method consists in determining vector or matrix equations having as solutions the axes components for the kinematic parameters 0, 1 and 2 in the reference system chosen.

The following symbols are used for systematization of mechanisms containing elementary cams:

$$C_C C_+ - P_C P_+$$

where:

C_C - the type of the lower coupling connecting the cam and the basis, being R for rotation coupling and T for translation coupling;

C_+ is the lower coupling between tapped and basis, being R for rotation coupling and T for translation coupling;

P_C is the profile of the cam near the contact point and it can be circle (C), line (D), point (P) or other curves noted in accordance with their nature;

P_+ shows the shape of the tapped near the contact point and it can be circle (C), line (D), point (P) or other curves noted in accordance with their nature.

The following cam mechanisms can be built:

RR – CC; RR – CD; RR – DC;

RT – CC; RT – DC; RT – CD.

We assume we have a mechanism with a rotation cam R, a circular cam C and a circular cam lever C.

In accordance with the positions and the angular velocity ω = const. of the cam, we shall determine the positions, velocities and angular accelerations of the cam lever. The following constructive data r, R_1, R_2, R, d and $\omega_{10} = \text{const.}$ are considered and presented in figure 1.

The assignment of coordinate systems results in two types of transformation matrices between coordinate systems: constant and variable. The transformation matrix between coordinate systems fixed at two different positions on the same link is constant. Transformation matrices relating the position and orientation of coordinate systems on different links include joint variables and thus they are variable [5].

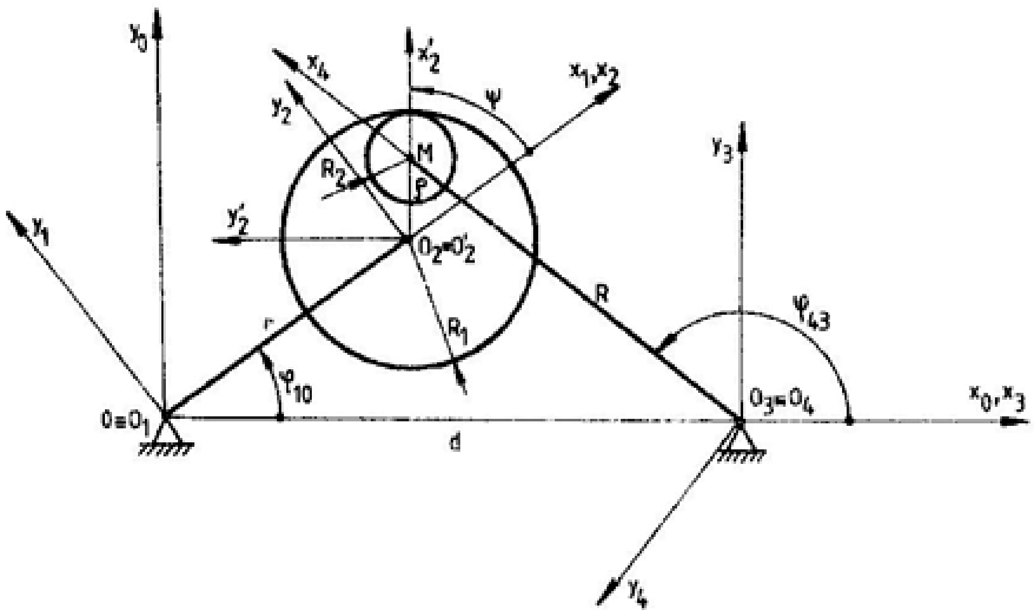

Fig. 1. Cam mechanism type RR-CC

The closing matrix equations are the following :

$$(r_M) = (r_{10}) + [a_{10}]^T (r_{21}) + [a_{10}]^T [a_{21}]^T (r_{2'2}) + \\ + [a_{10}]^T [a_{21}]^T [a_{2'2}]^T (\rho) \tag{1}$$

$$(r_M) = (r_{30}) + [a_{30}]^T (r_{43}) + [a_{30}]^T [a_{43}]^T (R), \tag{2}$$

where:

$$(r_{10}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; (r_{21}) = \begin{bmatrix} r \\ 0 \end{bmatrix}; (r_{2'2}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \tag{3.1}$$

$$(\rho) = \begin{bmatrix} \rho \\ 0 \end{bmatrix}; (r_{30}) = \begin{bmatrix} d \\ 0 \end{bmatrix}; \tag{3.2}$$

$$(r_{43}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; R = \begin{bmatrix} R \\ 0 \end{bmatrix}; \tag{3.3}$$

$$[a_{10}]^T = \begin{bmatrix} \cos\varphi_{10} & -\sin\varphi_{10} \\ \sin\varphi_{10} & \cos\varphi_{10} \end{bmatrix}; \\ [a_{2'2}]^T = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\varphi \end{bmatrix}; \tag{4}$$

$$[a_{30}]^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$[a_{43}]^T = \begin{bmatrix} \cos\varphi_{43} & -\sin\varphi_{43} \\ \sin\varphi_{43} & \cos\varphi_{43} \end{bmatrix}. \tag{5}$$

Replacing Eq. (4) in Eqs. (1) and (2) and taking into account the equations of the vector (r_M), the following system of projection equation is obtained [1],[2]:

$$\begin{cases} r\cos\varphi_{10} + \rho\cos(\varphi_{10} + \psi) = d + R\cos\varphi_{43} \\ r\sin\varphi_{10} + \rho\sin(\varphi_{10} + \psi) = R\sin\varphi_{43} \end{cases} \tag{6}$$

where:

- $\rho = R_1 - R_2$, if the roll of the cam lever moves toward the interior of the cam profile;
- $\rho = R_1 + R_2$, if the roll of the cam lever moves toward the exterior of the cam profile (Fig. 1).

In order to determine the velocity and the acceleration of the tapped, the relation is derived with time [4], [6]:

$$[a_{10}]^T (r_{21}) + [a_{10}]^T [a_{2'2}]^T (\rho) = \\ = (r_{30}) + [a_{43}]^T (R) \tag{7}$$

in which we have taken into account the expression of the matrices provided by Eqs. (3).

Therefore, the equation expressing the distribution of velocities is:

$$[a_{10}]^T [\omega_{10}](r_{21}) + [a_{10}]^T [\omega_{10}][a_{2'2}]^T (\rho) + \\ + [a_{10}]^{10} [a_{2'2}]^T [\omega_{2'2}](\rho) = \\ = [a_{43}]^T [\omega_{43}](R) \tag{8}$$

where:

$$[\omega_{10}] = \begin{bmatrix} 0 & -\omega_{10} \\ \omega_{10} & 0 \end{bmatrix} = \\ = \omega_{10}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \omega_{10}[u] \tag{9}$$

$$[\omega_{2'2}] = \begin{bmatrix} 0 & -\omega_{2'2} \\ \omega_{2'2} & 0 \end{bmatrix} = \\ = \omega_{2'2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \omega_{2'2}[u] \tag{10}$$

A system of two equations with two unknown variables $\omega_{2'2}$ and ω_{43}. is obtained.

$$[\omega_{43}] = \begin{bmatrix} 0 & -\omega_{43} \\ \omega_{43} & 0 \end{bmatrix} = \\ = \omega_{43}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \omega_{43}[u] \tag{11}$$

Through a second derivation with time of the Eq. (8) we obtain the equation for the accelerations distributions:

$$[a_{10}]^T [\omega_{10}]^2 (r_{21}) + [a_{10}]^T [\omega_{10}]^2 [a_{2'2}]^T (\rho) + \\ + 2[a_{10}]^T [\omega_{10}][a_{2'2}]^T [\omega_{2'2}](\rho) + \\ + [a_{10}]^T [a_{2'2}]^T [\omega_{2'2}]^2 (\rho) + \\ + [a_{10}]^T [a_{2'2}]^T [\varepsilon_{2'2}](\rho) = \\ = [a_{43}]^T [\omega_{43}]^2 (R) + \\ + [a_{43}]^T [\varepsilon_{43}](R) \tag{12}$$

$$[a_{10}]^T [\omega_{10}]^2 \{(r_{21}) + [a_{2'2}](\rho)\} + \\ + 2[a_{10}]^T [\omega_{10}][a_{2'2}]^T (\rho) + \\ + [a_{10}]^T [a_{2'2}]^T \{[\omega_{2'2}]^2 + [\varepsilon_{2'2}]\}(\rho) = \\ = [a_{43}]^T \{[\omega_{43}]^2 + [\varepsilon_{43}]\}(R) \tag{13}$$

The projection equations are determined, from which the angular accelerations $\varepsilon_{2'2}$ and ε_{43} are obtained, by using the matrix Eq. (13).

3. REPLACING MECHANISMS METHOD OF ANALYSIS

The method of replacing mechanisms requires the determination of the movement, the velocity and the acceleration of the driven element, in two ways: either knowing the profile of the cam in polar coordinates (which is also the equation of the cam lever‘s movement), or considering the curve of the cam lever’s profile and determining the cam lever’s movement depending on it [3],[7],[8].

In the first case it is about studying a function and its derives.

In the second case, to determine the movement of the cam lever replacing mechanisms are used.

The method is based on the equivalence theory of the general kinematic chains with fundamental chains: if the equivalent kinematic element has like dimension the distance between the curvature centre of the two profiles in contact, than the kinematic conduct of the two chains is identical.

4. CONCLUSIONS

In this work, the author presents the kinematic analysis of a particular cam mechanism by type RR-CC, using the matrices by form 3x3.

Using the matrix calculations represents a modern approach of the theoretical mechanics, giving new application opportunities in fields like: kinematic and dynamic analysis of mechanisms with bars, kinematic and dynamic analysis of kinematic chains for positioning (LCP) used in robots building, etc.

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