

## WORKFLOW NETS FOR FLEXIBLE MANUFACTURING SYSTEMS

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### Abstract

Since the sixties, the industry has begun an accelerated process of automation, which has led to the need for analysis, definition, modeling and management of the workflows for the manufacturing processes. A flexible manufacturing system (FMS) is an automatized system which includes automatic equipments for quick and correct operations, handling of products and materials and who includes computerized control for activities. Due to the complexity of these systems, the necessity to verify their correctness theoretically by mathematical methods before being implemented in practice is obvious. A workflow net (WF net) is a mathematical model, based on Petri nets, for workflow modeling. This paper wants to emphasize the advantages of using WF nets for describing FMSs and to show how this can be realized.

**Keywords:** workflow nets, flexible manufacturing systems, modeling, soundness

### 1. INTRODUCTION

Nowadays, workflow management systems are present in various fields such as: industry, e-commerce, banking, health-care, etc. This led to the idea of mathematical modeling of workflows. One of the easiest and most successful ways of specifying and modeling workflow systems is the one based on Petri net. A Petri net is a mathematical theory that can model the relations between conditions and events [1]. An advantage of modeling workflows with Petri Nets is the intuitive visual representation using graphs. A Petri net that models a workflow is called workflow net (WF net). At the beginning of the WF nets based approach, the focus was on controlling the execution of the system. Now, real systems must take into account the introduction of time or resource constraints, and priorities between tasks.

There are many approaches of FMSs modeling based on Petri nets [2-6], but, as far as we know, no FMS has been modelled by a WF net taking into account the previous constraints. A WF nets based approach is needed because there are resource-constrained workflow nets (RCWF nets) [7-9], priority workflow nets [10], multi-level workflow nets with resource constraints (mlRCWF nets) which are a compositions of standard workflow nets with shared resources [11].

It is necessary to demonstrate some properties that guarantee the correct execution of the tasks in a WF net. Such a property is called soundness. This property ensures the system execution without errors (deadlocks or livelocks). Soundness was formulated for classical WF nets and it was analyzed from the point of view of decidability and complexity [12-17]. This property was then extended to WF nets with resource constraints [8, 9], to WF nets with time constraints [11], and to WF nets with priorities [10].

Soundness of WF nets is decidable because it can be reduced to the boundness and liveness problems and these problems are decidable. If we add some constraints, the soundness property can be lost. It was proved that for priority WF nets soundness is undecidable [10]. The main objective of the researchers is to investigate the soundness property, and if it is undecidable, to identify subclasses of WF nets for which the soundness is decidable. Also, we want to identify those subclasses of Petri nets

for which soundness can be decided more efficiently. So far, it is known that the soundness problem of bounded WF nets is PSPACE-complete [15], weak soundness for bounded asymmetric-choice WF nets is co-NP-hard [14], soundness of 1-bounded acyclic workflow nets is co-NP-complete and weak soundness of 3-bounded acyclic asymmetric-choice workflow nets is co-NP-complete [17], for asymmetric choice WF-nets, if they are one-bounded or k-bounded ( $k > 1$ ), the soundness problem is co-NP-hard and the soundness is equivalent to the weak soundness for any acyclic asymmetric choice WF-nets [16].

For classical Petri nets the issue of modeling and analysis is also developed based on  $(\max, +)$  and MinMax algebra [18]. We consider that such an approach can be used also for WF nets.

This paper presents the basic definitions from WF nets theory and shows how this technique can be effectively used for describing and analyzing FMSs so that we can take advantage of the theoretical results obtained.

## 2. PRELIMINARIES

**Definition 2.1.** A Petri net is a tuple  $\Sigma = (P, T, F, W)$ , where  $P$  and  $T$  are two non-empty, finite sets of places and transitions,  $P \cap T = \emptyset$ ,  $F \subseteq (P \times T) \cup (T \times P)$  is the flow relation and  $W$  is the weight function  $W : (P \times T) \cup (T \times P) \rightarrow \mathbf{N}$ , where  $W(x, y) = 0$  iff  $(x, y) \notin F$ .

Usually, places denotes states and transitions denotes events.

A marking of  $\Sigma$  is a function  $M : P \rightarrow \mathbf{N}$  and it can be represents as a  $|P|$ -dimensional vector representing the number of tokens in each place.

For  $t \in T$  we define  $t^-, t^+ : P \rightarrow \mathbf{N}$ ,  $t^-(p) = W(p, t)$ ,  $t^+(p) = W(t, p)$ ,  $\Delta t(p) = t^+(p) - t^-(p)$ .

$t$  is enabled at a marking  $M$  (denoted by  $M[t]_{\Sigma}$ ), iff  $t^- \leq M$ . If  $t$  is enabled at  $M$ , then  $t$  may fire resulting a new marking  $M' = M + \Delta t$ . We write  $M[t]_{\Sigma}M'$ .

**Definition 2.2.** A workflow net (WF net) is a Petri net  $\Sigma$  with the following properties:

- 1)  $\Sigma$  has two special places  $i$  (called the input place of  $\Sigma$ ) and  $o$  (called the output place of  $\Sigma$ ),  $i$  has no input arcs, and  $o$  has no output arcs.
- 2) any node  $x$  in the graph of  $\Sigma$  is on a path from  $i$  to  $o$ .

The most important correctness criteria for WF nets is soundness. This property states that the system terminates its execution, at the end of the process the place  $o$  should be marked by exactly  $k$  tokens (if the execution started with  $k$  tokens in the initial place), all other places should be unmarked, and there are no dead tasks.

**Definition 2.3.** Let  $\Sigma$  be a WF-net and  $k \geq 1, k \in \mathbf{N}$ .  $\Sigma$  is called k-sound if for any  $M \in [M_{ki}]_{\Sigma}$  we have  $M_{ko} \in [M]_{\Sigma}$ , where  $M_{ki}$  is the initial marking with  $k$  tokens in the input place and all the other places empty and  $M_{ko}$  is the final marking with  $k$  tokens in the output place and all the other places empty.

A resource constrained WF net (RCWF net) is a workflow net whose set of locations is the union of the set of places form of base net with the set of places for resources. In a sound RCWF net the initial marking contains  $k$  tokens in the place  $i$  and some tokens in the resource places and the final marking contains  $k$  tokens in the place  $o$  and the same number of tokens in the resource places as they were at the beginning (all the other places are empty) (Fig.1).

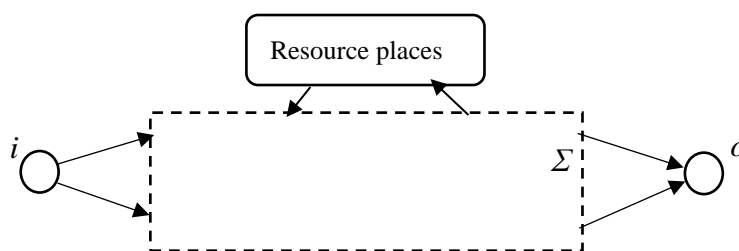


Fig.1. ARCWF net

A FMS is an automatized system based on: flexible machines, automated systems for handling material and products, computerized control for traffic control, ordering of pieces on machines, etc.

Although there are some modeling approaches of FMSs based on Petri nets since the nineties, there is no model that meets all the requirements of a real system with time and resources constraints, and priorities between tasks. Under certain assumptions, we believe that the behaviour of FMSs can be simulated using WF nets with the above type of constraints and, eventually, multilevel.

### 3. MODELING TECHNIQUE AND DISCUSSIONS

Because industrial processes require the use of various resources (devices, robots, human resources, etc.), we consider necessary their modelling by RCWF nets.

To illustrate the modelling of a FMS using WF nets we present a real example of an assembling system (Fig.2). We consider that two products a and b are processed by two devices X and Y. After their processing, the products a' and b' are obtained; a' and b' are assembled together by the robot Z resulting the new product c. In the corresponding RCWF net (Fig. 2) the locations have suggestive names. The transitions  $t_{i1}$  and  $t_{i2}$  represent the taking over of products a and b by the processing systems. The transitions  $t_a$  and  $t_b$  realize the transformation of the initial products into the products a' and b', using the devices X and Y represented as resource-places. The transition  $t_c$  combines the two products into the final product c using the robot Z, represented by a resource-place and releases the resources X and Y. The transition  $t_o$  transport the product c to the output location and releases the Z resource. The RCWF net in figure 2 is k-sound and it guarantees the correctness of the FMSs modelled.

In the real world, product processing requires more operations which must be performed in a certain order. In this case, we can use WF nets with resources and priorities. Priorities can be introduced as relation  $\rho$ , where  $t \rho t'$  denotes the fact that  $t'$  has priority over  $t$  [10]. Unfortunately the soundness property for priority WF nets is undecidable. For this reason, we suggest designing the WF net so that the transitions fire in a certain order, without the need to use the priorities.

Taking into account the fact that an operation is not performed instantaneously, but it needs a certain time to complete, we can associate a time duration function to the WF net,  $\delta: T \rightarrow N$ , which associates to each transition an amount of time.

Also, if the manufacturing process requires multiple stages, we can implement multi-level WF nets.

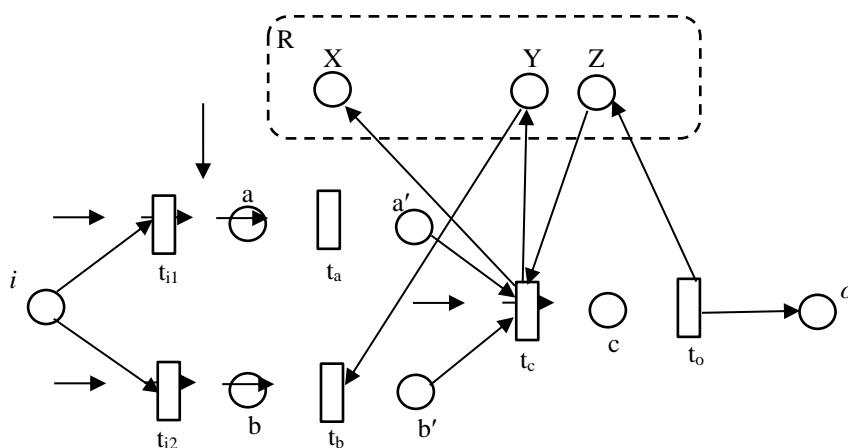


Fig 2. A RCWF net for an assembling system

### 4. CONCLUSIONS

WF nets are a suitable tool for modeling and analyzing FMSs properties. Although there are some modeling approaches of FMSs based on Petri nets since the nineties [1-6], there is no model of WF net that includes all the requirements of a real system with time, priorities and resources constraints

eventually, multilevel. This is why RCWF nets with time constraints and priorities are a useful tool for modeling FMSs. The results obtained regarding the soundness property in the case of RCWF nets can be transferred to the corresponding FMSs.

Obtaining other results on the complexity of some subclasses of WF nets would be of great interest both for theoretical and practical use.

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