INCREASE THE LOAD OF LOSS OF STABILITY FOR THE PILLARS OF LARGE-OPENING HALLS

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ABSTRACT

At the ground floor halls with an opening or more, the forces that are transmitted through the pillars due to gravitational loads (snow, and her weight) have high values. The gravitational difference that is transmitted to the pillars increases the greater the opening between the pillars is. In this Article we analyse the pillars of halls that are articulated recessed, because they have the advantage that the joints do not transmit bending moments. In the case of halls with large openings or halls rehabilitated or strengthened, to increase the critical load of loss of stability can increase the moment of inertia of the pillars from the recessed to the articulated end. In this Article, a parametric numerical study is presented, which establishes the critical load at a pillar with a variable section, depending on the length of the area where the moment of inertia of the slat has been increased.

KEYWORDS: Monte Carlo method, moments of inertia of flat surfaces, probabilistic algorithms

1. Introduction

Industrial ground floor halls (Fig. 1) are structural systems in which the axial forces transmitted through the pillars are very high. The first cause of significant values of axial forces is the very large opening between the pillars which leads to large gravitational afferent. Another important cause of increased axial loads in pillars is the existence of bridge cranes. A simple and effective solution to increase the critical load is to increase the moment of inertia of the pillars, starting from the stapling of the pillars in the good foundation ground to the articulated end where the gravitational input from the wraparound flows. Increasing the moment of inertia to newly designed structures is done from the design phase, and the area that is enlarged for optimization is established from the design phase of the structures. For the existing structures to which the destination is changing, due to new loads must be designed mansions of different lengths and with certain moments of inertia to be applied to the pillars of the hall in areas established by calculation. Application of these mansons will lead to increased critical loads of loss of stability [1].

2. Theoretical and numerical model

The pillar shown in Fig. 2 is considered to be the static diagram of an articulated-inlay pillar from an industrial hall. For the pillar with the constant cross-section of Fig. 1 the equation of the deformed mean fiber is written on the deformed form, and by solving this equation the critical charge is determined [2].

Fig. 1. Industrial ground floor halls
For a current section of the pillar in Fig. 1, the variation expression of the bending moment at distance $x$ is:

$$M(x) = P \cdot v - H \cdot x$$

From the differential equation of the approximate deformed mid fibre results:

$$\frac{d^2 v}{dx^2} = \frac{M(x)}{EI};$$
$$\frac{d^2 v}{dx^2} + \frac{P \cdot v}{EI} = \frac{H \cdot x}{EI};$$
$$k^2 = \frac{P}{EI}$$

The solution of the nonhomogeneous second order differential equation with constant coefficient is:

$$v(x) = v_\circ(x) + v_p(x):$$
$$v(x) = C_1 \sin(kx) + C_2 \cos(kx);$$
$$v_p(x) = Cx \rightarrow \frac{P \cdot v_p}{EI} = \frac{H \cdot x}{EI};$$
$$v_p = \frac{H}{P} \cdot x$$

The general solution of the differential equation is:

$$v(x) = C_1 \sin(kx) + C_2 \cos(kx) + \frac{H}{P} \cdot x;$$
$$v''(x) = C_1 k \cos(kx) - C_2 k \sin(kx) + \frac{H}{P}$$

The constants of integration are determined from the conditions of existence of the deformed form as the form of equilibrium:

$$x = 0;$$
$$v(0) = 0;$$
$$C_2 = 0;$$
$$x = l;$$
$$v(l) = 0;$$

After applying the limit conditions, the homogeneous system of equations is obtained:

$$\begin{cases}
C_1 \sin(kl) + \frac{H}{P} l = 0; \\
C_1 \cos(kl) + \frac{H}{P} = 0.
\end{cases}$$

The unknowns of the system of equations (5) are $C_1$ and $H/P$, the homogeneous system of equations has non-trivial solutions if the determinant of the coefficients is zero.

$$\begin{vmatrix}
\sin(kl) & 1 \\
\cos(kl) & 1
\end{vmatrix} = 0;$$
$$\sin(kl) - kl \cos(kl) = 0;$$
$$\tan(kl) = kl;$$
$$kl = 4.493;$$
$$(kl)^2 = \left(\frac{\pi}{0.699}\right)^2;$$
$$P_{cr} = \frac{\pi^2 EI}{(0.699l)^2}$$

To analyse pillars with a variable moment of inertia along the bar, consider the pillar of Fig. 3, which consists of two zones, one with the moment of inertia $I$ and the other with the moment of inertia $2I$.

To determine the critical load of the column for different ratios between the area with the moment of inertia $I$ and $2I$, a Matlab program was created that determines the critical load for loss of stability by the finite element method. The program is based on the stability equation, which for a structure has the form [2]:

$$\det([K_{el}] - \lambda[K_e]) = 0.$$ 

where:

$[K_{el}]$ is the matrix of elastic stiffness of
structures that is also used in the first-order calculation of structures

$$[K_g]$$ is the geometric matrix of the structure.

By solving the stability equation, the eigenvalues $$\lambda_{(i)} = 1, 2, 3 \ldots$$ are obtained. The minimum value $$\lambda_{min}$$ represents the multiplication factor that is applied to the axial force determined from a first-order calculation.

There are cases when the increase of the moment of inertia of the column is difficult to achieve from the embedment towards the free end of the column, because the muff to be applied must be designed so that the foundation takes the capable moment of the muff. To avoid this inconvenience, the muff can be applied inside the pillar. This avoids the end areas which are areas whose grips are already calculated and conformed. Fig. 4 shows a pillar with a muff applied in the middle.

For different values of the length of the muff, the critical force for loss of pillar stability can be determined. By increasing the length or the moment of inertia for the muff, the critical force can be increased up to the required value. The solution with the muff in the middle is more practical and easier to achieve than the solution with a muff that starts from the recess. The main advantage of a reinforced hall with a muff in the middle is represented by the fact that the muff can be applied without dismantling the hall and stopping the technological processes taking place inside the hall. For ease of assembly and execution, the muff is made of two half-pieces that are welded together. To fix the muff on the pillar, the ends of the muff are welded to the pillar of the hall.

![Fig. 4. pillar with a muff applied in the middle](image)

### 3. Results and conclusions

To highlight the increase of the critical force in the bar with variable section from the embedment to the free end Fig. 3, a pillar with a length of 8 m articulated-embedded made of steel with $$E = 2.1 \cdot 10^5$$ MPa, was analyzed, with the ring cross-section with outer diameter $$D = 100$$ mm and the inner diameter $$d = 50$$ mm.

For different values of the ratio between the length of the zone with moment $$2I$$ and the length of the zone with inertia moment $$I$$, the ratio of the critical force at the bar with variable section to the critical force at the bar with constant section is shown in Fig. 4.

![Fig. 5. The critical force at the bar with variable section to the critical force at the bar with constant section](image)

From the analysis of the results obtained, it follows:

As the length of the zone with inertia moment $$2I$$ increases, an increase in the values of the critical force of loss of stability can be observed.

If the entire pillar is considered with the moment of inertia $$2I$$ or $$I$$, the critical force calculated with the relation of Euler is obtained, which validates the calculation program and implicitly the results from the numerical study.

In the case of the pillar with a muff in the middle of Fig. 4, a parametric study was carried out on how the critical force increases depending on the ratio between the length of the muff $$2I$$ with the moment of inertia $$2I$$ and the bar of length $$I$$ with the moment of inertia $$I$$. The study was carried out on an
8 m long hinged-recessed pole made of steel with $E = 2.1 \cdot 10^5$ MPa, having an annular cross-section with outer diameter $D = 100$ mm and inner diameter $d = 50$ mm.

**Fig. 6.** Ratio of critical force bar with variable section to bar with constant section

Based on the results represented in Fig. 6, it follows:
- with the increase of the area with moment of inertia $2I$, the critical force of loss of stability increases.
- for the case when the length of the sleeve is the same as the length of the bar, Euler's relationship for calculating the critical load is obtained, a fact that validates the numerical program for calculating the simulation of the loss of stability.

**References**
