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MATHEMATICAL MODELING OF THE CASTING PROCESS OF THE SECOND MERGER

Marian-Iulian NEACȘU
"Dunarea de Jos" University of Galati, Romania
e-mail: mneacsu@ugal.ro

ABSTRACT

The paper shows the method of making the equations of the mathematical model that describes the dependence of several mechanical and casting properties on the chemical composition of the cast irons of the second fusion and on the casting conditions.

The chemical composition taken into account refers to the contents of C, Si and Mn and the casting conditions analysed were: casting temperature, superheating time and superheating temperature.

After performing the specific mathematical calculations, the specific mathematical equations were obtained that describe the dependence of the studied properties according to the parameters of the casting process taken into account.

KEYWORDS: casting temperature, fluidity, gray cast iron, mathematical model

1. Introduction

The global energy crisis determines the use of material resources that include low manufacturing, processing and exploitation costs. Thus, the existence and accessibility on the market of materials with increasingly high mechanical performances is taken into account.

The development and casting of cast iron is not a simple but a complex process. This process depends on many factors that have an important influence on the mechanical properties of the cast iron parts thus made [1, 2].

Cast irons are second fusion cast irons usually produced in arc furnaces or induction furnaces. These cast irons, if they are developed according to the standards in force and if they are subsequently applied to certain specific treatments, can compete in properties even with some steels [2].

For the most precise management of the elaboration and casting processes, it is useful to resort to the establishment of mathematical models based on which to find the most efficient agreement between the values of the parameters of the two processes and the values of the properties that these pieces must achieve [3].

The construction of a mathematical model for any process is conditioned by the fulfilment of some contrary requirements that must be met in the most balanced way possible. On the one hand, the developed mathematical model must be quite simple, be a representation of the real system with a certain degree of abstraction, and on the other hand, it must be a fairly faithful representation of the system it models [4].

In the complete understanding of reality, it is often necessary to know, understand and master the connections between two or more phenomena, quantified by variables.

This actually requires being able to build and then use for forecasting, the so-called statistical models or regression models, these being models that describe the existing correlation between any two variables and, in particular, between a variable and time [5].

Most of the time, the foundation of these models is based on a large volume of data and this is where the software packages designed to assist the forecast calculations prove their usefulness.

In regression modeling we start from the following situation: given two variables X and Y, studied in a population A, the question arises whether between the two variables, respectively between the phenomena described by them, there is a certain dependence called correlation [4, 5].

A rigorous substantiation of the existence of a correlation and then of the model that describes the correlation, also called a regression model, can be

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done based on the calculation and interpretation of some statistical indicators [5, 6].

In the case of some regression model, the forecast based on the model is the more truthful, the better the model is chosen to fit the data and the smaller the forecast horizon.

Regression models are part of the category of stochastic (statistical) models, in which all explanatory factors of a phenomenon, which do not find their place directly in the model, appear accumulated in the form of a random variable called error.

A variable Y (the output parameter) that quantifies the studied phenomenon can be explained by regression on one or more explanatory factors X (input parameters). If we have two or more explanatory (predictive) factors, X_1, X_2, ..., X_p, then the regression is called multiple and the corresponding model will be [4, 6]:

$$Y = f(X_1, X_2, ..., X_p) \quad (1)$$

2. Experimental conditions

The casting properties and mechanical properties of some cast irons produced in electric induction furnaces were investigated as a function of chemical composition and casting conditions.

The cast iron was developed in the induction furnace having as raw materials carbon steel waste, superheated to 1550 °C. The modification process was done in the silicocalcium pot.

The mathematical model that was proposed to create is based on the active experiment method that uses a number of data obtained by performing programmed experiments.

The equation of the mathematical model is of the form of equation (1) and written unfolded has the form:

$$Y=b_0+x_1 \cdot b_1+x_2 \cdot b_2+ x_3 \cdot b_3+x_4 \cdot b_4+ x_5 \cdot b_5 + x_6 \cdot b_6, \quad (2)$$

The independent variables of the proposed mathematical model are the following:

x_1 - carbon content, in %;
x_2 - silicon content, in %;
x_3 - manganese content, in %;
x_4 - casting temperature, in °C;
x_5 - overheating time, in °C;
x_6 - overheating temperature, in °C.

The dependent variables of the proposed mathematical model are:

Y_1 - fluidity, in mm;
Y_2 - linear contraction, in %;
Y_3 - initial expansion, in %;
Y_4 - tendency of hot crack formation, in %;
Y_5 - bleaching tendency, in %;
Y_6 - HB type hardness, in MPa;
Y_7 - tensile strength, in MPa.

The chemical composition, the content of C, Si and Mn, of the cast irons subjected to the experiments is shown in Table 1.

Table 1. *C, Si and Mn content of the cast irons under investigation*

	%C	%Si	%Mn
Min	2.5	1.5	0.2
Max	3.5	2.5	0.6

The experimental conditions (pouring temperature, superheating temperature and superheating time) are those from Table 2.

Table 2. *Experimental conditions*

Nr. crt.	Pouring temperature, °C	Overheating time, in min.	Overheating temperature, in °C
1	1450	40	1580
2	1400	30	1530
3	1350	20	1480

After conducting the experiments, the basic level, the upper level, the lower level and the range of variation of the values for the six parameters considered in order to create the equations of the mathematical model for the researched process were established. The upper level was denoted by (+1), the lower level by (-1), the base level by (0), and the variation range by $\triangle xi$. Table 3 illustrates these notations with their associated values.

Table 3. *Initial calculation data*

The level of variation	X_1	X_2	X_3	X_4	X_5	X_6
Top Level (+1)	3.5	2.5	0.6	1450	40	1580
Base Level (0)	3.0	2.0	0.4	1400	30	1530
Lower Level (-1)	2.5	1.5	0.2	1350	20	1480
The range of variation $\triangle xi$	0.5	0.5	0.2	50	10	50

Since k (the number of variables) is equal to six, 2^6 = 64 experiments should be performed, but since the values of six coefficients have to be calculated, I made a fractional program with a replica of 1/8 of 64, namely 8 experiments with the conditions: $1 = x_1 x_3 x_5 = x_1 x_4 x_6 = x_2 x_3 x_6 = x_2 x_4 x_5 = x_1 x_2 x_3 x_4 = x_1 x_2 x_5 x_6 = x_3 x_4 x_5 x_6$, (3).

The experimental matrix for the 8 experiments is shown in Table 4.

The values of the coefficients resulting from the calculations are presented in Table 5.

Table 4. *Matrix of the experiment*

Nr. exp.	X_0	X_1	X_2	X_3	X_4	X_5	X_6	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
1	+1	-1	-1	-1	-1	-1	-1	163	1.8	0.015	83	100	429	35.0
2	+1	+1	-1	+1	-1	-1	+1	573	1.4	0.057	31.5	48	235	23.5
3	+1	+1	-1	-1	+1	+1	-1	1210	1.0	0.170	7.0	60	248	18.3
4	+1	-1	+1	+1	-1	+1	-1	450	1.5	0.050	52.5	100	415	29.4
5	+1	-1	+1	-1	+1	-1	+1	540	1.5	0.015	33.5	74	277	30.0
6	+1	+1	+1	+1	+1	-1	-1	1033	1.0	0.015	2.0	59	235	15.0
7	+1	+1	+1	-1	-1	+1	+1	840	1.5	0.040	15.0	54	212	13.6
8	+1	-1	-1	+1	+1	+1	+1	455	2.8	0.030	55.5	100	415	14.4

Table 5. *Values of regression coefficients*

Property	b_0	b_1	b_2	b_3	b_4	b_5	b_6
Y_1	645.5	256	57.75	-30.25	151.5	80.75	-56
Y_2	1.56	-0.34	-0.19	0.11	0.01	0.14	0.24
Y_3	0.045	0.025	-0.015	0.015	0.005	0.02	-0.017
Y_4	35	-21.25	-9.25	-0.4	-10.5	-3.5	-1.1
Y_5	74.3	-19.1	-2.62	2.37	-1.12	4.12	-5.37
Y_6	308.3	-75.8	-23.5	16.8	-14.5	14.3	-23.5
Y_6	22.9	-5.3	-0.9	-2.3	-3.5	-4.0	-2.5

The equations of the mathematical model whose coefficients were calculated are:

$$Y_1 = 645{,}5 + 256 \cdot X_1 + 57{,}75 \cdot X_2 - 30{,}25 \cdot X_3 + 151{,}5 \cdot X_4 + 80{,}75 \cdot X_5 - 56 \cdot X_6 \quad (4)$$

$$Y_2 = 1{,}56 - 0{,}34 \cdot X_1 - 0{,}19 \cdot X_2 + 0{,}11 \cdot X_3 + 0{,}01 \cdot X_4 + 0{,}14 \cdot X_5 + 0{,}24 \cdot X_6 \quad (5)$$

$$Y_3 = 0{,}045 + 0{,}025 \cdot X_1 - 0{,}015 \cdot X_2 + 0{,}015 \cdot X_3 + 0{,}005 \cdot X_4 + 0{,}020 \cdot X_5 - 0{,}017 \cdot X_6 \quad (6)$$

$$Y_4 = 35 - 21{,}25 \cdot X_1 - 9{,}25 \cdot X_2 - 0{,}4 \cdot X_3 - 10{,}5 \cdot X_4 - 3{,}5 \cdot X_5 - 1{,}1 \cdot X_6 \quad (7)$$

$$Y_5 = 74{,}3 - 19{,}1 \cdot X_1 - 2{,}62 \cdot X_2 + 2{,}37 \cdot X_3 - 1{,}12 \cdot X_4 + 4{,}12 \cdot X_5 - 5{,}37 \cdot X_6 \quad (8)$$

$$Y_6 = 308{,}3 - 75{,}8 \cdot X_1 - 23{,}5 \cdot X_2 + 16{,}8 \cdot X_3 - 14{,}5 \cdot X_4 + 14{,}3 \cdot X_5 - -23{,}5 \cdot X_6 \quad (9)$$

$$Y_7 = 22{,}9 - 5{,}3 \cdot X_1 - 0{,}9 \cdot X_2 - 2{,}3 \cdot X_3 - 3{,}5 \cdot X_4 - 4{,}0 \cdot X_5 - 2{,}5 \cdot X_6 \quad (10)$$

where: $x_1 = \frac{C-3{,}0}{0{,}5}$, $x_2 = \frac{Si-2{,}0}{0{,}5}$, $x_3 = \frac{Mn-0{,}4}{0{,}2}$, $x_4 = \frac{T_t-1400}{50}$, $x_5 = \frac{\tau_{supr}-30}{10}$, $x_6 = \frac{T_{supr}-1530}{50}$

3. Conclusions

From the analysis of the equations of the developed mathematical model, the following conclusions result:

- with the increase of carbon and silicon content, the fluidity of the alloy increases and the linear contraction, the tendency to form hot cracks and the tendency to bleach decrease;

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- hardness and tensile strength decrease as the carbon and silicon content increases;
- from equation (4) it can be seen that the casting temperature and the overheating temperature have a strong influence on the fluidity of the alloy, being directly proportional to it;
- the longer the overheating time, the more it leads to a decrease in the strength characteristics of the cast alloy, according to equations (9) and (10);
- the superheating temperature has a not very big but similar influence to the superheating time on the mechanical characteristics (mechanical strength and hardness) as shown by equations (9) and (10).

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