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THE IMPACT OF PRESSURE ANGLE AND TOOTH ROOT ON THE MODIFIED ELLIPTICAL GEARS BENDING STRESS AND FATIGUE LIFE

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ABSTRACT

Noncircular gears are receiving growing attention due to their ability to deliver variable transmission ratios and to optimize motions in specialized mechanical and industrial applications. Unlike circular gears, noncircular gears experience continuous variation in pressure angle and load distribution throughout the meshing cycle, leading to non-uniform stress states among teeth. This study explores the impact of pressure angle and tooth root radius on the bending stress and fatigue life of modified elliptical gears. Finite Element Analysis (FEA) is conducted on three representative teeth positioned at varying distances from the gear centre, highlighting the asymmetric loading characteristic of specific profiles. The tooth root radius is varied, showing a clear reduction in maximum stress with the increased fillet radius size. Then, by varying the rack cutter angle, it is shown that higher angles reduce stress concentration. The results reveal the critical influence of tooth position and geometry on the fatigue vulnerability in noncircular gear design.

KEYWORDS: noncircular gears, bending stress, fatigue life

1. Introduction

Noncircular gears are mechanical components capable of transmitting rotary motion with a variable velocity ratio, offering unique advantages compared to traditional circular gears. Instead of having a constant radius, their shape follows a controlled variation that allows the gear to impose specific motion patterns during rotation. This property makes them particularly useful in specialized applications such as textile machinery with intermittent feeding cycles, variable-stroke pumps [1], moulding presses [2] with optimized force distribution, and robotic linkages [3] that require non-uniform angular velocities. Among noncircular geometries, elliptical gears are among the most widely used due to their relatively simple profiles and ease of integration into existing mechanisms.

Unlike circular gears, where each tooth is loaded under identical conditions, noncircular gears present a continuously changing loading scenario. Teeth stresses are affected by the pressure angle, contact point, and effective force arm throughout the rotation, which is determined by their position. Consequently, identifying the most critical tooth and understanding how design parameters affect stress levels are essential for reliable operation and fatigue performance.

In this paper, we analyse these two significant geometric parameters, the rack cutter pressure angle and the fillet radius at the tooth root, which are examined concerning the distribution of stress [4-6] during gear generation in modified elliptical gears. Using Finite Element Analysis (FEA), we analyse three distinct teeth located at different positions on the gear, as their loading conditions vary significantly due to the nonuniform geometry. The goal is to identify which tooth is subjected to the highest stress and how design adjustments can improve performance and reduce the risk of fatigue failure [7-9] in noncircular gear applications.

2. Gear design using the Gielis' supershape for the pitch curve

One of the key aspects in designing non-circular gears lies in defining the geometry of the driving gear's pitch curve. Given the virtually unlimited range of shapes that can be generated, along with the ease of adjusting a predefined contour, the Gielis

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superformula [10] is proposed as a suitable tool for describing the non-circular centrode. This equation offers a flexible parametric form capable of modeling a wide variety of closed curves, including modified elliptical shapes.

$$r_0(\varphi) = \left(\left| \frac{1}{a} \cos \frac{k\varphi}{4} \right|^{n_2} + \left| \frac{1}{b} \sin \frac{k\varphi}{4} \right|^{n_3} \right)^{-\frac{1}{n_1}} \quad (1)$$

where φ is the polar angle; r_0 - the polar coordinate of the "unit" centrode; *a, b* - the conventional ellipse semi-axes lengths, inscribed or circumscribed about the centrode; *k* – a real positive parameter that introduces the curve's rotational symmetry; n_2, n_3 – real positive parameters that lead to "polygonal" shapes; n_1 – a real positive and non-zero parameter that modifies the curve's geometry with respect to the linearity/convexity of the sides and the sharpness/flattening of the corners [11].

When the parameter $k = 4$, the superformula leads to modified elliptical shapes, suitable as gears pitch curves.

A fundamental parameter in gear design is the pressure angle, defined as the angle between the line of action and the common tangent to the pitch circles of two meshing gears. It is the parameter that determines the direction of force transmission of gear teeth during contact. A larger pressure angle will create a stronger tooth shape with greater load-carrying capacity, as it thickens the base of the tooth and reduce bending failure.

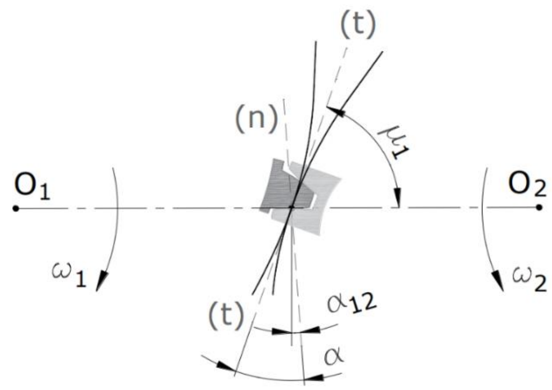

Fig. 1. *Pressure angle components*

However, increasing the pressure angle also raises the radial component of the transmitted force, leading to higher bearing loads and potential vibration or noise issues. A smaller pressure angle improves smoothness and reduces friction but weakens the tooth root and increases stress concentration.

In noncircular gears, the pressure angle can vary dynamically during rotation, making it even more critical to control through design. Additionally, since the pressure angle is defined by the generating tool (such as the rack cutter), it directly influences the shape of the tooth flanks and the gear's mechanical performance.

The pressure angle is defined by equation:

$$\alpha_{12} = \mu \pm \alpha_c - \frac{\pi}{2} \quad (2)$$

$$\mu = \tan^{-1}\left(\frac{r_0(\varphi)}{r_0'(\varphi)}\right) \quad (3)$$

where α_c is the angle of the rack cutter, $r_0(\varphi)$ is the radius value at the contact point, and $r_0'(\varphi)$ is the derivate of the Gielis function.

In order to maintain the pressure angle within acceptable limits, Litvin [12] recommends limiting the pressure angle α_{12}, which should remain within the range $-50° \leq \alpha_{12} \leq 50°$.

Exceeding the above range may result in unfavourable contact conditions and excessive lateral forces between the meshing teeth, which can compromise the smoothness and reliability of power transmission. Moreover, when a specific transmission ratio is required, it is often nearly impossible to adjust the pressure angle directly, since the geometry of the pitch curve is already constrained. In such instances, the optimal solution is to alter the generating tool angle (α_c).

Based on the theoretical considerations presented above, a specific case study was developed using the Gielis superformula to define the pitch curve geometry of the driving gear. The noncircular centrode was designed using the following parameters: $a = 1.3$, $b = 1$, $k = 4$, $n_1 = 2.5$, $n_2 = 2.5$,

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$n_3 = 2$. The “unit centrode” was further scaled to fit as the gear pitch curve, choosing 28 teeth and a module of 1 mm for the gear geometry; the addendum and dedendum were set to standard values of 1 mm and 1.25 mm, respectively, ensuring proper tooth engagement and clearance. The gear face width was set to 3 mm. Prior to conducting the finite element simulations, the variation of the pressure angle along the generated pitch curve was examined to ensure that all values remained within the range recommended by Litvin. The variation is presented in Fig. 2 considering an initial α_c = 20°. This validation step was essential to confirm the feasibility of the selected noncircular profile.

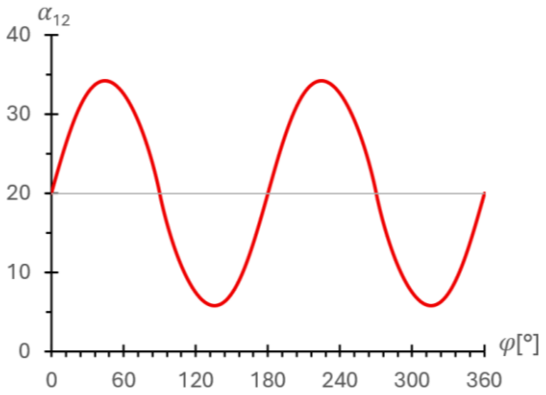

***Fig. 2.** Variation of pressure angle*

The noncircular gear was generated using the enveloping theory [13], where the tooth profile results from the envelope of a single rack cutter tooth moving in pure rolling motion along the pitch curve. For the generation of the analysed cases, three rack cutters were used, each with a different pressure angle: 18°, 20°, and 22° (Figure 3). These values were chosen to investigate how the tool geometry influences the resulting tooth profile and the stress distribution in the gear. The generation of the tooth profile using a 20° rack cutter is shown in Figure 4.

The gear generation was simulated in AutoCAD using an AutoLISP code. To reduce the number of elements during finite element analysis, only a segment of the driving and driven gears was modeled.

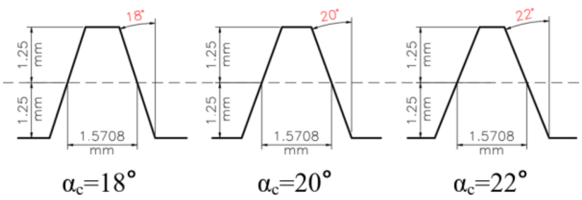

Fig. 3. Rack cutter design

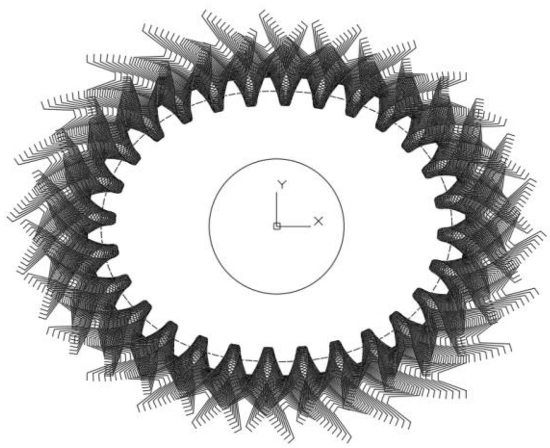

***Fig. 4.** Tooth profile generation using the enveloping theory*

Three teeth were selected in key positions along the pitch curve to be analysed in detail. Figure 5 shows the selected contact points along with the angles at which contact takes place.

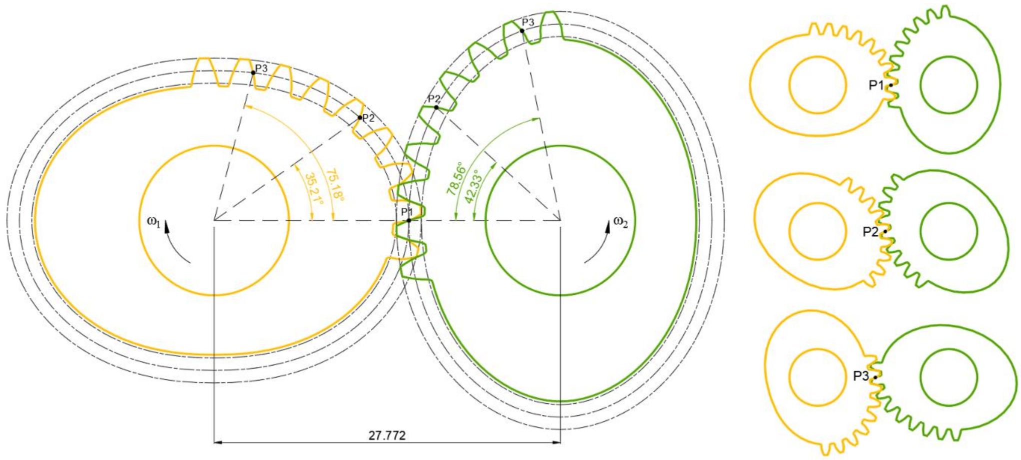

***Fig. 5.** Segments of generated gears and selected contact points during meshing*

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The tooth root fillet radius obtained through the enveloping theory is approximately 0.15 mm. However, with the advent of modern manufacturing technologies such as EDM and CNC machining, the enveloping theory often serves only as a virtual tool for gear profile generation. In practice, the obtained tooth shape can also be optimized or adjusted after preliminary profile synthesis.

For this reason, another objective of the study was to investigate the potential reduction of bending stress by increasing the root fillet radius. Simulations were also performed for enlarged radii of 0.20 mm and 0.25 mm (Figure 6), allowing the evaluation of how root geometry modifications affect stress concentration and fatigue resistance.

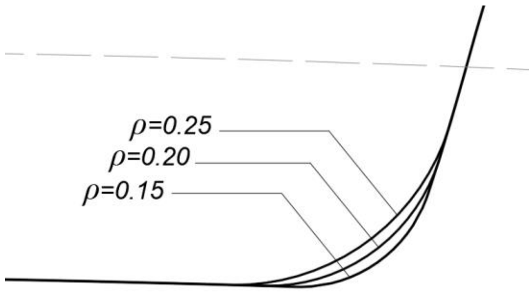

Fig. 6. *Variation of the tooth root fillet radius*

After the geometry was finalized, the model was exported to Ansys and prepared for finite element analysis. A coarse mesh was applied globally, with local refinement in two critical areas: the contact zone and the tooth root region, where bending stresses are expected to be highest. In these regions, the element size was set to 0.05 mm, resulting in a total of approximately 33,000 elements and 42,000 nodes, with slight variations depending on the analysed case.

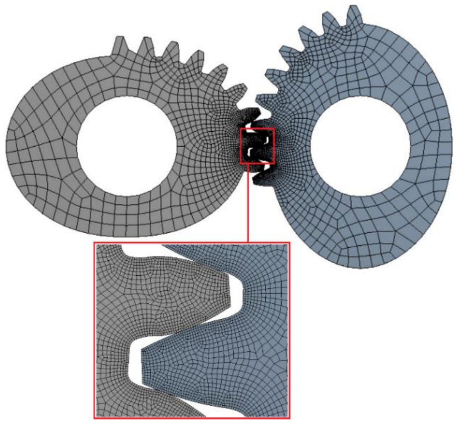

Fig. 7. *Mesh discretization*

The driven gear was fully constrained, while a torque of 2,000 Nmm was applied to the driving gear to simulate loading conditions.

3. Results and discussions

Three teeth were chosen in different areas of the gear to be analysed individually. The results are grouped based on the root radius (0.15 mm, 0.20 mm, 0.25 mm) and cutter angles (18°, 20°, 22°), so that we can easily compare how each change influences stress levels. For each case, the maximum stress value at the tooth base was recorded.

First, a total of nine simulations were carried out, three for each selected contact point. In each case, the tooth root fillet radius was varied between 0.15 mm, 0.20 mm, and 0.25 mm to observe how this parameter influences the bending stress at the base of the tooth. The results for the first case, where the rack cutter angle is 20° and the tooth root fillet radius is 0.15 mm, are shown in Figure 8. The stress distribution is presented for all three contact points, both on the driving gear and the driven gear.

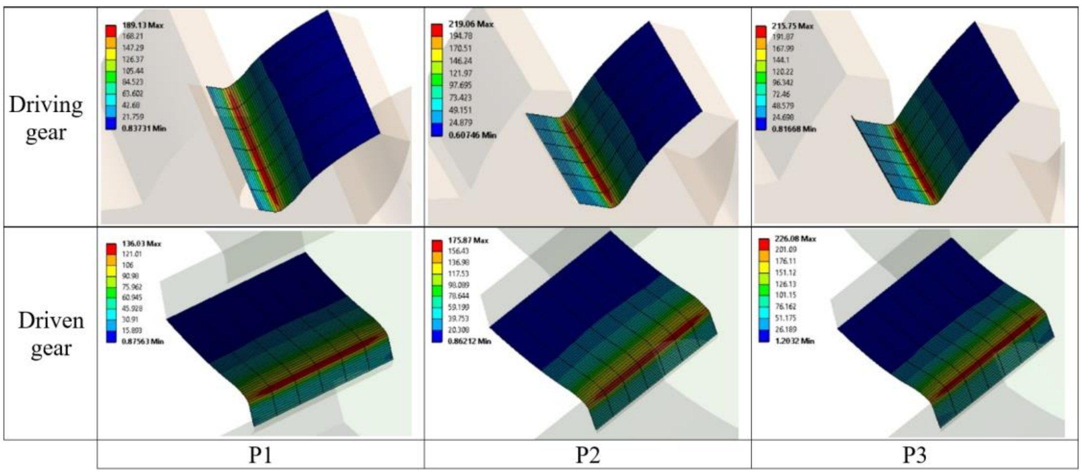

Fig. 8. *Von-Mises stress on the tooth root, $\alpha c = 20°$, $\rho = 0.15$ mm*

The remaining results for αc = 20°, with root radii of 0.20 mm and 0.25 mm, are shown in Figures 9 and 10, for the driving gear and driven gear, respectively, allowing for a direct comparison between the two. The chart displays the maximum von Mises stress values at all three contact points (P1, P2, and P3) and all radii values.

For the driving gear (Figure 9), the highest stress levels occur at P2, closely followed by P3, with maximum values reaching 219.06 MPa and 215.75 MPa, respectively, when ρ = 0.15 mm. As the root radius increases to 0.25 mm, the stress at these points drops by 13.46%, confirming the effectiveness of fillet radius optimization in reducing bending loads.

For the driven gear (Figure 10), a similar trend is observed. The highest value appears at P3, with a maximum of 226.08 MPa for the smallest radius. Again, increasing ρ decreases the stress significantly, with a reduction of 9.69% at ρ = 0.25 mm.

These results show that the critical tooth position on the driven gear is P3.

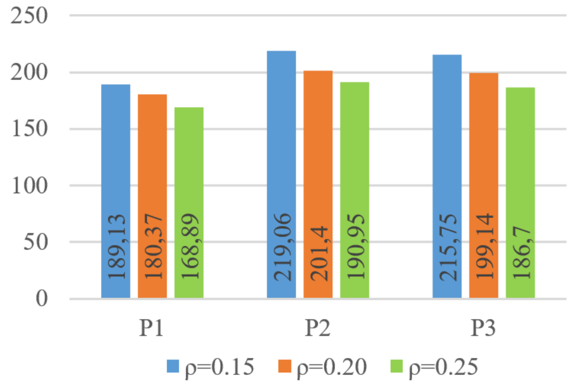

***Fig. 9.** Maximum von Mises Stress at Tooth Root – Driving Gear*

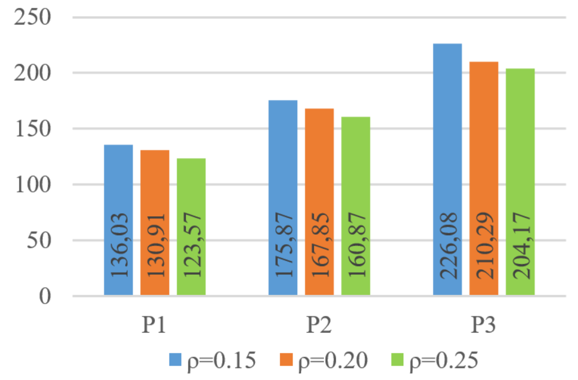

***Fig. 10.** Maximum von Mises Stress at Tooth Root – Driven Gear*

After identifying that the lowest stress values occurred for a root fillet radius of 0.25 mm, a second set of simulations was performed to study the influence of the rack cutter pressure angle (αc). New gear geometries were generated using αc = 18° and 22°.

The results of these simulations are shown in Figure 11 and Figure 12, where the maximum von Mises stress at the tooth root is compared for all three contact points (P1, P2, and P3), both on the driving and the driven gear. Reducing the rack cutter angle to 18° resulted in an increase in stress values of up to 5%, while increasing the angle to 22° led to a reduction in stress of up to 7% compared to the original case at 20°. This confirms the positive effect of using a slightly larger pressure angle on bending stress reduction, due to the increased thickness at the tooth root.

Overall, the results indicate that both tooth root radius and the rack cutter angle play a crucial role in determining stress concentration at the tooth's base. While the direct impact of radius is clear, an impact on pressure angle can also lead to a notable improvement in gear strength.

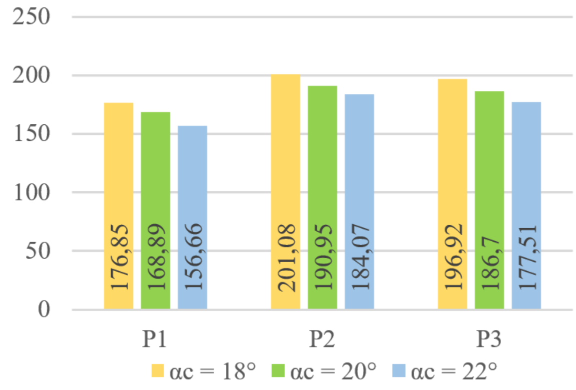

***Fig. 11.** Maximum von Mises Stress at Tooth Root – Driving Gear*

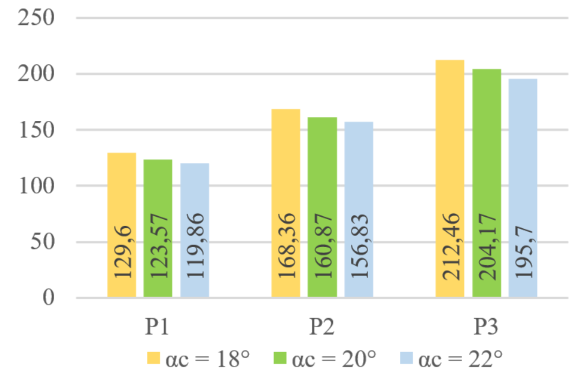

***Fig. 12.** Maximum von Mises Stress at Tooth Root – Driven Gear*

Since gear teeth are typically subjected to repeated cyclic loading during operation, evaluating fatigue behavior is essential for predicting long-term durability and avoiding premature failure. Crack formation can occur over time due to localized stress concentrations at the tooth root, even if the static stress levels remain below the material's yield strength. For this reason, the next step in the study focuses on analysing the potential for fatigue failure.

The number of load cycles was calculated for each gear based on the applied torque during rotation. This analysis was carried out for three different configurations, corresponding to rack cutter pressure angles of 18°, 20°, and 25°, to observe how tooth geometry influences fatigue life. The results are presented in Figure 13. The focus was on identifying the variation in expected life between the driving and driven gears for each case. For a given torque level, the gear generated with αc = 22° consistently supports a higher number of cycles compared to the other configurations. In contrast, the geometry with αc = 18° exhibits the shortest fatigue life at all load levels.

This confirms that a larger pressure angle not only reduces bending stress but also enhances fatigue resistance, likely due to the thicker tooth root and lower stress concentration.

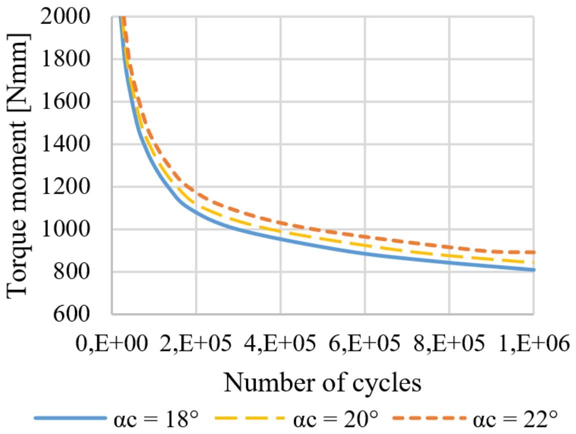

***Fig. 12.** Fatigue Life Curves*

For example, at a torque of 1000 Nmm, the fatigue life increases by approximately 58% when the pressure angle is increased from 18° to 22°, rising from 2.98 × 10^5 to 4.73 × 10^5 cycles. In general, fatigue life increases by between 30% and 60% across the two extreme cases analysed (αc = 18°, αc = 22°), highlighting the significant influence of the rack cutter angle on the gear's durability.

4. Conclusions

The results of this study confirm that both the tooth root fillet radius and the rack cutter pressure angle have a significant impact on the stress distribution and fatigue life of modified elliptical gears. Increasing the root radius consistently results in a decrease in bending stress while improving fatigue resistance, particularly at the most heavily loaded tooth positions.

However, when designing noncircular gears, one must also consider the geometric constraints imposed by the pitch curve. In some cases, it may be necessary to reduce the pressure angle to keep it within the limits recommended by Litvin (±50). While this adjustment guarantees proper meshing, it often results in higher stress levels and reduced fatigue life.

On the other hand, a higher rack cutter angle can be effective in minimizing bending stress and maximizing gear life. Yet, this approach has its own drawbacks, as it tends to increase the radial force component, which may result in increased vibrations and noise, as well as increasing bearing loads.

Both low- and high-pressure angles have their advantages and disadvantages. The optimal solution depends on the specific application, design constraints, and performance requirements. A balanced approach is required to ensure both mechanical strength and smooth operation in real-world conditions.

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