Mechanical Testing and Diagnosis
ISSN 2247 – 9635, 2014 (IV), Volume 2, 5-15

FRAME ROTATION EFFECT ON STATIC PERFORMANCE OF SHORT JOURNAL BEARINGS: COUPLE STRESS FLUID MODEL

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ABSTRACT

The objective of this paper is to predict the performance of a rotor bearing system, lubricated with couple stress fluid accounting the cross effect of velocities. Modified Reynolds equation and the Stokes constitutive equation have been obtained. Narrow bearing approximations have been used to obtain the solutions in the closed form. The effects of couple stress, frame rotation and the viscosity-density effects have been numerically obtained. The obtained results are consistent with the physical situation of the problem.

Keywords: friction, hydrodynamic, journal, thin film

INTRODUCTION

Theory of squeeze film has, so far, been playing a very important role in the field of engineering, in practical situations, such as lubrication of machine elements, lubrication in human body at synovial joints etc., [1, 2, 3]. Thus, a considerable attention has been paid on it by scientists and researchers [4], showing that there is a good agreement between the theoretical and experimental results for a Newtonian squeeze film behavior, between fixed and rotating annulii. Afterwards, a large number of researchers studied and concluded on different types of fluids and their behavior as a squeeze film. In 1981, Banerjee et al. [5] pointed out that almost all the physical systems are under effect of rotation - though it may be very small and they have extended the classical theory of lubrication [6]. Banerjee et al., and Gupta et al. [7, 8, 9] have shown that, in certain situations, the qualitative properties of the bearing system may be different and they obtained a certain class of fundamental solutions of the generalized Reynolds equation, which are not allowed in the classical Reynolds theory [6]. In 1987, Gupta et al. [8] elaborated a model to show theoretically the effect of small rotations in squeeze film journal bearing and have shown that the obtained results are in a good agreement with the practical results.

On the other hand, the theory of non-Newtonian polar fluids has also been the center of attention for the researchers, due to their increasing use in industrial machine elements based on the rheological behavior because of the presence of additives, suspensions and long chain polymers. Among the polar fluid theories, the couple stress fluid theory developed by Stokes [10], which considers couple stresses in addition to the classical Cauchy stress, has been of much interest for the researchers, from a long time. It is the generalization of the classical fluid theory, which allows for polar effects, such as the presence of couple stresses and body couples. Linear shearing stress and shearing rate relationships do not exist for such lubricants. Stokes discussed the fluid theory in detail in his treatise [11]. Afterwards, the squeeze film lubrication of couple stress fluid has been studied by Lin [12] and Ramanaiah [13] and they observed an increase in load carrying capacity.

Lin [12] has recently investigated the effect of couple stress lubricant on static characteristic of a rotor bearing and analyzed the problem under the assumptions of negligible shear stress between rotor and bearing system, which, in practice, has a measurable effect. Also, the effect of small rotation on the performance of a bearing system cannot be overlooked. Therefore, the journal bearing under squeezing film condition along with the shearing stress will behave differently and need to be investigated under realistic conditions.

Hence, in order to investigate the problem under the said realistic physical condition, as in the classical theory developed by Reynolds [6] and extended by Banerjee, Gupta and Kavita [7, 8, 9] in 1982 and onward, the modified Reynolds equation has been obtained, using the microcontinuum theory for lubricants containing substructures [14]. The interaction of microcontinuum theory developed by Stokes [10] for lubricants with polar effects that is couple stress, body couples and the non-symmetric tensors has been used to develop the generalized Reynolds equation. The constitutive equations of an incompressible fluid with couple stress and small rotations [10, 15, 16] are:

$$\nabla \bar{V} = 0 \tag{1}$$

$$\rho \frac{D\bar{V}}{Dt} = -\nabla p + \rho \bar{F} + \frac{1}{2}\rho \nabla \times C + \mu \nabla^2 \bar{V} - \eta \nabla^4 \bar{V} + 2\rho\left(\bar{V} \times \bar{\Omega}\right) \tag{2}$$

where the vectors $\bar{V}$, $\bar{F}$, C and $\bar{\Omega}$ represent the velocity, the body force per unit mass, the body couple per unit mass and the rotation, respectively; ρ is the density of the fluid, p is the pressure, μ is the shear viscosity and η is the new material constant standing for the couple stress fluid property.

CONSTITUTIVE EQUATIONS AND BOUNDARY CONDITIONS

The physical configuration of a journal bearing is shown in Fig. 2.1. Consider a layer of fluid, which is kept rotating at a constant rate. Let Ω be the angular velocity of the frame rotation about y axis. The lubricant in the system is taken to be Stokes couple stress fluid. The body forces and the body couples are assumed to be absent. Under the assumptions of hydrodynamic lubrication, applicable to a thin film, as used by Pinkus and Sternlitch [17] and *Singh* [18], the field equations governing the motion of an incompressible fluid given in Cartesian co-ordinate system are:

$$\frac{\partial p}{\partial x} = 2p\Omega w + \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \tag{3}$$

$$\frac{\partial p}{\partial y} = 0 \tag{4}$$

$$\frac{\partial p}{\partial z} = -2p\Omega u + \mu \frac{\partial^2 w}{\partial y^2} - \eta \frac{\partial^4 w}{\partial y^4} \tag{5}$$

These equations are solved under the following boundary conditions, for u and w:

$$u = 0 \quad \text{at} \quad y = 0 \text{ and } h \tag{6}$$

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x} \quad \text{at} \quad y = 0 \text{ and } h \tag{7}$$

$$w = 0 \quad \text{at} \quad y = 0 \text{ and } h \tag{8}$$

$$\mu \frac{\partial^2 w}{\partial y^2} = \frac{\partial p}{\partial z} \quad \text{at} \quad y = 0 \text{ and } h \tag{9}$$

where u and w are the velocity components in x and z directions, respectively and h is the film thickness between the journal bearing system.

ANALYSIS

The simplification of equations eq. (30) and eq. (5) gives rise to differential equations of order 8, as below:

$$\left(\frac{\eta^2}{2\rho\Omega}\right)\frac{\partial^8 u}{\partial y^8} - \left(\frac{\mu\eta}{\rho\Omega}\right)\frac{\partial^6 u}{\partial y^6} + \left(\frac{\mu^2}{2\rho\Omega}\right)\frac{\partial^4 u}{\partial y^4} + \left(2\rho\Omega u\right) = -\frac{\partial p}{\partial z} \tag{10}$$

$$\left(\frac{\eta^2}{2\rho\Omega}\right)\frac{\partial^8 u}{\partial y^8} - \left(\frac{\mu\eta}{\rho\Omega}\right)\frac{\partial^6 u}{\partial y^6} + \left(\frac{\mu^2}{2\rho\Omega}\right)\frac{\partial^4 u}{\partial y^4} + \left(2\rho\Omega w\right) = -\frac{\partial p}{\partial x} \tag{11}$$

that cannot be solved with only four boundary conditions, eqs. (6-7) for u and eqs. (8-9) for w and, therefore, a schematic iterative technique has been applied to find the solutions of the equations (3) and (5) under the boundary conditions (6) through (9), for velocity components u and w. The step-wise procedure is as under.

METHODOLOGY

Step-I

In the first iterative approximation, the solution for u and w is considered as under, in which only the second order velocity derivative has been considered while neglecting the cross effects:

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - yh\right) \tag{12}$$

$$w = \frac{1}{2\mu}\frac{\partial p}{\partial z}\left(y^2 - yh\right) \tag{13}$$

Step-II

Using the above velocities u and w as initial solution, the next iterative solution is as under, considering the cross effects along with the second order variation in velocity:

$$u = \frac{\partial p}{\partial x}\left[\frac{1}{2\mu}\left(y^2 - yh\right) + \frac{1}{\eta}\left(\frac{y^4 - 2y^3 + yh^3}{4!}\right)\right] + \frac{2\rho\Omega}{\mu\eta}\frac{\partial P}{\partial z}\left(\frac{y^6 - 3y^5h + 5y^3h^3 - 3yh^5}{6!}\right) \tag{14}$$

$$w = \frac{\partial p}{\partial z}\left[\frac{1}{2\mu}\left(y^2 - yh\right) + \frac{1}{\eta}\left(\frac{y^4 - 2y^3 + yh^3}{4!}\right)\right] - \frac{2\rho\Omega}{\mu\eta}\frac{\partial P}{\partial x}\left(\frac{y^6 - 3y^5h + 5y^3h^3 - 3yh^5}{6!}\right) \tag{15}$$

which appear to be a better solution than the earlier one.

Step-III

In order to further improve the solution obtained for u and w, substituting their values up to the second order term of velocity, the next improved solution is as:

$$u=\frac{\partial p}{\partial x}\left[\frac{1}{2\mu}\left(y^2-yh\right)+\frac{\mu}{\eta^2}\left(\frac{y^6-3y^5h+5y^3h^3-3yh^5}{6!}\right)-\frac{(2\rho\Omega)^2}{\mu\eta^2}\left(\frac{y^{10}-5y^9h+30y^7h^3-126y^5h^5+255y^3h^7-355yh^9}{10!}\right)\right]+\frac{\partial P}{\partial z}\left[\frac{2\rho\Omega}{\mu\eta}\left(\frac{y^6-3y^5h+5y^3h^3-3yh^5}{6!}\right)+\frac{4\rho\Omega}{\eta^2}\left(\frac{y^8-4y^7h+14y^5h^3-28y^3h^5+17yh^7}{8!}\right)\right] \quad (16)$$

$$u=\frac{\partial p}{\partial z}\left[\frac{1}{2\mu}\left(y^2-yh\right)+\frac{\mu}{\eta^2}\left(\frac{y^6-3y^5h+5y^3h^3-3yh^5}{6!}\right)-\frac{(2\rho\Omega)^2}{\mu\eta^2}\left(\frac{y^{10}-5y^9h+30y^7h^3-126y^5h^5+255y^3h^7-355yh^9}{10!}\right)\right]+\frac{\partial P}{\partial x}\left[\frac{2\rho\Omega}{\mu\eta}\left(\frac{y^6-3y^5h+5y^3h^3-3yh^5}{6!}\right)+\frac{4\rho\Omega}{\eta^2}\left(\frac{y^8-4y^7h+14y^5h^3-28y^3h^5+17yh^7}{8!}\right)\right] \quad (17)$$

3.2 Particular Cases

Case-I

If the bearing system consists of a Newtonian fluid $(\eta=0)$, the above said solution is identical with Gupta and Banerjee [8], as under:

$$u=\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2-yh\right)-\frac{2\rho\Omega}{\mu^2}\frac{\partial P}{\partial z}\left(\frac{y^4-2y^3h+yh^3}{4!}\right)+\ldots \quad (18)$$

$$w=\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(y^2-yh\right)-\frac{2\rho\Omega}{\mu^2}\frac{\partial P}{\partial x}\left(\frac{y^4-2y^3h+yh^3}{4!}\right)+\ldots \quad (19)$$

Case-II

If the bearing system consists of Stokes couple stress fluid, the above solution is identical with that presented by Lin [12] as under, up to a good approximation to the usual solution obtained by Lin:

$$u=\frac{Uy}{h}-\frac{1}{\eta}\frac{\partial P}{\partial x}\left(\frac{y^4-2y^3h+yh^3}{4!}\right)-\frac{\mu}{\eta^2}\frac{\partial P}{\partial x}\left(\frac{y^6-3y^5h+5y^3h^3-3yh^5}{6!}\right)+\ldots \quad (20)$$

$$w=-\frac{1}{\eta}\frac{\partial P}{\partial z}\left(\frac{y^4-2y^3h+yh^3}{4!}\right)-\frac{\mu}{\eta^2}\frac{\partial P}{\partial z}\left(\frac{y^6-3y^5h+5y^3h^3-3yh^5}{6!}\right)+\ldots. \quad (21)$$

Hence, the developed technique for obtaining the cross effects of this problem under the defined boundary conditions is logically and physically correct to a good approximation because the direct solution of the equations (3) and (5) cannot be obtained because of the lack of additional boundary conditions due to 8-degree partial differential equation in u and w.

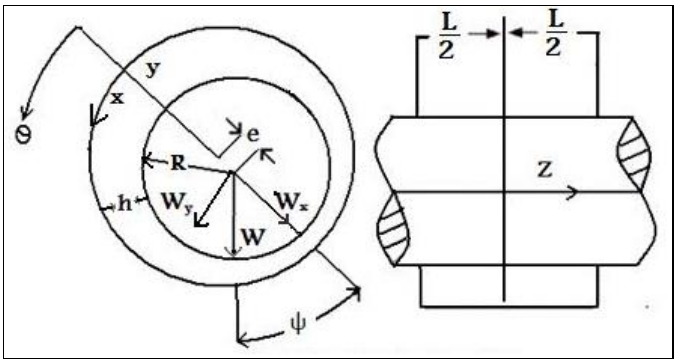

Fig. 2.1. Physical configuration

Now, replacing the velocity components for u and w in equations (16) and (17), respectively, in the continuity equation (1) and integrating with respect to y with the boundary conditions, equation (22-23)

$$v = 0 \qquad \text{at} \qquad y = 0 \tag{22}$$

$$v = -\frac{dh}{dt} \qquad \text{at} \qquad y = h \tag{23}$$

the modified Reynolds equation is finally derived in equation (24):

$$\begin{aligned} &\frac{\partial}{\partial x}\left[\left\{h^3 + \frac{17}{1680}\frac{\mu^2}{\eta^2}h^7 - \frac{13.086364}{30240}\frac{(2\rho\Omega)^2}{\eta^2}h^{11}\right\}\frac{\partial P}{\partial x}\right] \\ &+\frac{\partial}{\partial z}\left[\left\{h^3 + \frac{17}{1680}\frac{\mu^2}{\eta^2}h^7 - \frac{13.086364}{30240}\frac{(2\rho\Omega)^2}{\eta^2}h^{11}\right\}\frac{\partial P}{\partial z}\right] \\ &+\frac{\partial}{\partial x}\left[\left\{\frac{17}{28}\frac{(2\rho\Omega)}{6!\mu\eta}h^7 - \frac{31}{9}\frac{(4\rho\Omega)}{8!\eta^2}h^9\right\}\frac{\partial P}{\partial z}\right] \\ &+\frac{\partial}{\partial z}\left[\left\{\frac{17}{28}\frac{(2\rho\Omega)}{6!\mu\eta}h^7 - \frac{31}{9}\frac{(4\rho\Omega)}{8!\eta^2}h^9\right\}\frac{\partial P}{\partial x}\right] = -12\mu\frac{dh}{dt} \end{aligned} \tag{24}$$

Under assumptions of having a short bearing, $\frac{\partial}{\partial x} << \frac{\partial}{\partial z}$, the equation (24) reduces to:

$$\begin{aligned} &\frac{\partial^2 P}{\partial z^2}\left[h^3 + \frac{17}{1680}\frac{\mu^2}{\eta^2}h^7 - \frac{13.086364}{30240}\frac{(2\rho\Omega)^2}{\eta^2}h^{11}\right] \\ &+\frac{\partial P}{\partial z}\left[\frac{17}{240}\frac{(2\rho\Omega)}{\eta}h^6 - \frac{31}{1680}\frac{(2\rho\Omega)}{\eta^2}\mu h^8\right]\frac{dh}{dx} = -12\mu \end{aligned} \tag{25}$$

The non-dimensional modified Reynolds equation is obtained as:

$$A\frac{\partial^2 P}{\partial z^2} - B\frac{\partial P}{\partial z} = -48\lambda^2\cos\theta \tag{26}$$

where the superscript * has been dropped for simplicity and

$$A = h^3 + \frac{17}{1680}\frac{h^7}{l^4} - \frac{13.086364}{30240}\frac{M^2h^{11}}{l^4} \tag{27}$$

$$B = -\left(\frac{31}{10080}\frac{Mh^3}{l^4} - \frac{17}{120}\frac{Mh^6}{l^2} \right)\lambda\varepsilon\sin\theta \tag{28}$$

Solving the equation (25) for pressure P, with the condition of zero pressure at the bearing ends, i.e. $P = 0$ at $Z = \pm\frac{1}{2}$, the non-dimensional film pressure is obtained as under:

$$P = -\frac{c}{2B}\left[1 + 2z\right] + \frac{c}{2B}\left[\frac{Exp\left(\frac{Bz}{A}\right) - Exp\left(-\frac{B}{2A}\right)}{\sinh\left(\frac{B}{2A}\right)} \right] \tag{29}$$

where

$$C = -48\lambda^2\cos\theta \tag{30}$$

Since the rotational parameter M is small, ignoring the third and the higher power of M and using narrow journal bearing approximation to calculate the pressure, the simplified pressure P is:

$$P = \frac{C}{2A}\left[\left(z^2 - \frac{1}{4}\right) + \frac{B}{3A}\left(z^3 + \frac{1}{8}\right) + \frac{B^2}{12A^2}\left(z^4 - \frac{1}{16}\right) \right] \tag{31}$$

BEARING CHARACTERISTICS

Once the film pressure is determined from equation (31), the bearing characteristics can now be obtained as follows:

1.1. Load capacity :

The load capacity can be calculated integrating the film pressure acting on the journal rotor. The component of load along x (W_x) and y (W_y), the perpendicular to the center line, and load carrying capacity W are given by:

$$W_x = W.\cos\psi = -R\int_{\theta=0}^{\theta=\pi}\int_{z=-\frac{L}{2}}^{z=\frac{L}{2}} P\cos\theta dz d\theta \tag{32}$$

$$W_y = W.\sin\psi = R\int_{\theta=0}^{\theta=\pi}\int_{z=-\frac{L}{2}}^{z=\frac{L}{2}} P\sin\theta dz d\theta \tag{33}$$

$$W = \sqrt{W_x{}^2 + W_y{}^2} \tag{34}$$

where ψ is the altitude angle, defined by $\psi = \tan^{-1}\left(W_y / W_x\right)$.

Taking a non-dimensional load capacity $W^* = \frac{WC^2}{(d\varepsilon/dt)\mu R^3 L}$, the components of load carrying capacity in a non-dimensional form can be expressed as:

$$W_x = W^*.\cos\psi = -\int_{\theta=0}^{\theta=\pi}\int_{z=-\frac{1}{2}}^{z=\frac{1}{2}} P^*\cos\theta dz^* d\theta \tag{35}$$

$$W_y = W^*.\sin\psi = \int_{\theta=0}^{\theta=\pi} \int_{z=-\frac{1}{2}}^{z=\frac{1}{2}} P^* \sin\theta dz^* d\theta \tag{36}$$

The non-dimensional load capacity W^* can now be evaluated as:

$$W^* = \sqrt{W_x^{*2} + W_y^{*2}} \tag{37}$$

RESULTS AND DISCUSSION

In this paper, to account the effect of couple stress and frame rotation, a modified generalized Reynolds equation is derived. The effects of couple stress and rotation have been simultaneously considered for studying the variations of different bearing performance properties e.g. pressure, load carrying capacity, friction parameter etc. of the bearing system. Earlier researchers [12, 13] have not considered the effect of rotation when discussing the effects of couple stress on the bearing performance while Gupta et al. [8] considered the rotation in their investigation, but without the couple stress effect.

In the present study, an emphasis has been made to investigate the problem considering the simultaneous effect of small rotation and couple stress together with the variation in pressure along the squeezing direction on the performance of a short journal bearing. In order to discuss the effect of rotation, a dimensionless parameter M has been introduced [9]. Since M is a function of the rotation Ω, the bearing clearance c, the fluid density ρ and the fluid viscosity μ, it can be identified as the interaction of the fluid property, bearing geometry and bearing performance. For a particular fluid, a small value of *M* may be either due to small rotations or to the small bearing clearance, or both, and a larger value of *M* similarly depends on the rotation as well as on the bearing clearance; but, for no rotation, the value of M is being taken as zero. To study the effect of couple stress, the parameter l - a fluid property dependent has been used. Since l has dimension of length, the dimensionless parameter $l^*(=l/c)$ - a fluid and bearing property dependent, have been introduced.

In this process, numerical results have been obtained from the pressure equation (31), the load capacity equations (34) to (36), the friction parameter equation (40). Since in practice, the length to diameter ratio (λ) of a short journal bearing is preferred to be small, the value for λ is suitably taken as 0.3 throughout the discussion. The numerical results for the bearing properties under discussion have been obtained for couple stress parameter $l^* = 0.2, 0.4, 0.6, 1.0$ and eccentricity ratio $\varepsilon = 0.2,\ 0.4,\ 0.6,\ 0.8$, shown in figures 4.1 to 4.3 for pressure, figures 4.3 to 4.6 for load capacity, 4.7 and 4.8 for altitude angle and figures 4.9 to 4.10 for friction parameter.

To establish the results for the effect of couple stress, the variation of film pressure and load carrying capacity of the bearing due to couple stress, have been discussed without rotation. Further, to analyze the performance and behavior of the bearing under small rotation, the results have been considered for different values of the rotation parameter M, varying from 0 to 0.15.

Figure 4.1 shows the variation of the normalized pressure $\wp\left[= P^*/P_l\right]$ as a variation with respect to the circumferential angle *θ*, varying from 90° to 180° for the couple stress parameter $l^* = 0.2, 0.4, 0.6$, considering the bearing without rotation $(M = 0)$ at the mid plane $z^* = 0$ and an eccentricity ratio $\varepsilon = 0.6$. It is observed that the normalized pressure increases with the circumferential angle θ and reaches a maximum at $\theta = 180^o$.

The pressure below $\theta = 90^o$ has not been considered because of its small value. Moreover, the normalized pressure increases with the increase in couple stress at a particular angle θ, which is consistent to Lin formulation [12].

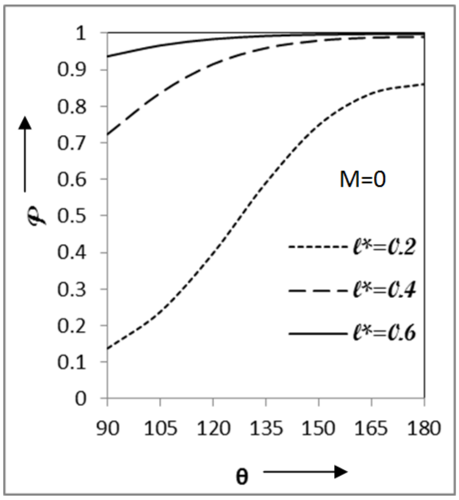

Fig. 4.1. Variation of Normalized pressure pressure as variation of θ for couple stress parameter at no rotation

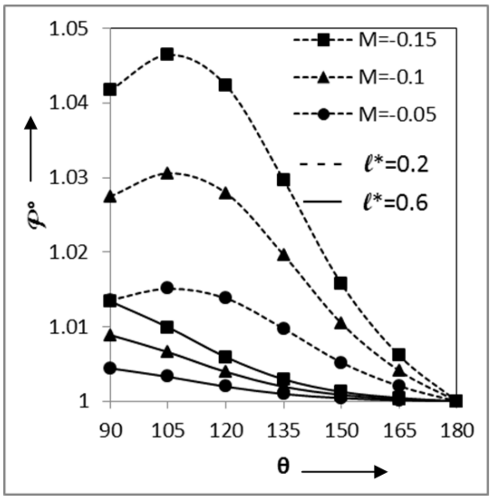

Fig. 4.2 Variation of Normalized as variation of θ for rotation parameter.

Figure 4.2 shows the variation of the relative pressure $\wp^o \left[= P^*/P_m\right]$ with respect to the circumferential angle θ, for different values of the rotation parameter M and couple stress parameter $l^* = 0.2,\ 0.6$ at $z^* = 0$ and an eccentricity ratio $\varepsilon = 0.6$. It is observed that the normalized film pressure is higher for higher values of M and for each nonzero value of M, the normalized film pressure is higher in comparison to the case without rotation. It is also observed that the effect of rotation is more dominant for lower values of l^* and decreases with the increase in the value of l^*.

Figure 4.3 shows the variation of the normalized pressure $\wp^o \left[= P^*/P_m\right]$ versus the dimensionless bearing coordinate $z^*\left[-4, 4\right]$, for different values of the rotation parameter M and the couple stress parameter l^*, at an eccentricity ratio $\varepsilon = 0.6$ and $\theta = 120^o$. Again, it is observed that the normalized film pressure is higher for the higher values of M whereas the nature of the curves shows the relative variation of the pressure to the pressure in absence of rotation because as M tends to 0, $\wp^o$ tends to 1**.**

Figure 4.4 shows the normalized load carrying capacity $W\left(= W^*/W_l\right)$ as a function of the bearing eccentricity ratio ε, for different values of the couple stress parameter $l^* = 0.2,\ 0.4,\ 0.6$, in the absence of rotation. It is observed that, as a result of increase in pressure, the load capacity increases with the increase in couple stress, which agrees with the results obtained by Lin [12].

Figure 4.5 shows the normalized load carrying capacity $W^a\left(= W^*/W_M\right)$ as a function of the eccentricity ratio ε for different values of the rotation parameter M at $l^* = 0.2,\ 0.4$. It is observed that the variation in the load capacity with the rotation follows the same pattern as that of the pressure variation with rotation, as shown in Fig. 4.2. As a consequence of the variation of the pressure, the load capacity increases with the increase in the value of rotation and for each value of M. Further, the load carrying capacity of the bearing is higher than the load capacity when the bearing is operated without rotation.

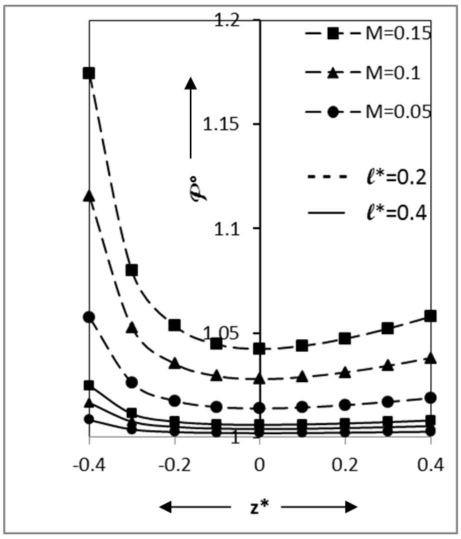

Fig. 4.3. Variation of Normalized pressure as variation of Z* for couple stress parameter

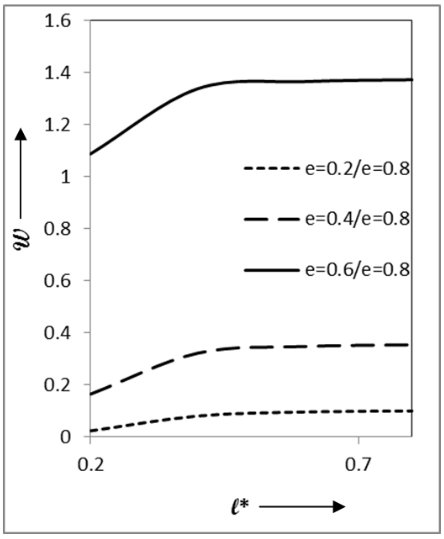

Fig. 4.4. Variation of Normalized load capacity as variation couple stress parameter at no rotation

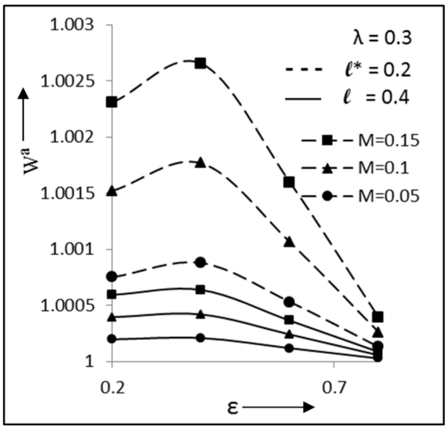

Fig. 4.5 Variation of Normalized load capacity as variation of eccentricity ratio for rotation parameter

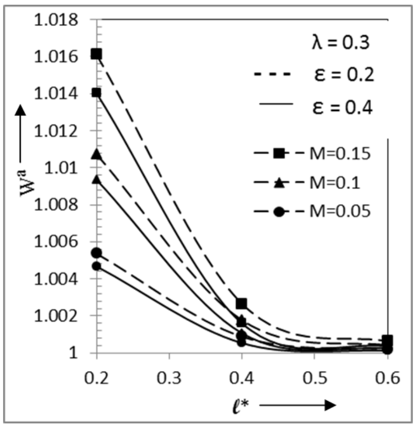

Fig. 4.6. Variation of Normalized load capacity as variation of couple stress parameter for rotation parameter

Figure 4.6 shows the normalized load capacity $W^a\left(=W^*/W_M\right)$ as a function of the couple stress parameter l^* for various values of the rotation parameter M, at an eccentricity ratio $\varepsilon = 0.2, 0.4$, respectively. It is observed that, for each value of ε and l^*, the variation of the normalized load capacity with rotation again agrees, the same as discussed earlier. It is also observed, from both the Figures 4.5 and 4.6, that the effect of the rotation on the load carrying capacity bearing is more dominant for lower values of l^*.

CONCLUSION

In the present theoretical study, the cross effect due to small rotation leads to much nearer to the realistic situation in the analysis, as well as its performance. This will increase the safety factor while designing such bearings.

NOMENCLATURE BEARING CLEARANCE

F_r Friction parameter $\frac{f\,R}{c}$.

F_{r_m} Friction paramete $M=0$.

F_{r^o} Normalized Friction parameter $\frac{F_r}{F_{r_m}}$.

h^* $\frac{h}{c}=1+\varepsilon\cos\theta$.

ℓ Couple stress parameter $\sqrt{\frac{\eta}{\mu}}$.

ℓ^* Dimensionless couple stress parameter $\frac{\ell}{c}$

L Bearing length.

M Frame rotation parameter $\left(\frac{2\rho\omega c^2}{\mu}\right)$

P^* Dimensionless pressure $\left(\frac{Pc^2}{\mu R^2(d\varepsilon/dt)}\right)$

P_ℓ Dimensionless pressure at $\ell^*=1$.

P_M Dimensionless pressure at $M=0$.

$\wp, \wp^o$ Normalized pressure; $\frac{P^*}{P_\ell}$, $\frac{P^*}{P_M}$.

R Radius of Journal.

W^* Dimensionless load capacity given by $\left(\frac{Wc^2}{\mu R^3\,L(d\varepsilon/dt)}\right)$.

W_ℓ Dimensionless load capacity at $\ell^*=1$.

W_M Dimensionless load capacity at $M=0$.

W Normalized load capacity $\left(\frac{W^*}{W_\ell}\right)$.

W^a Normalized load capacity $\left(\frac{W^*}{W_M}\right)$.

z^* $\frac{z}{L}$.

ε $\frac{e}{c}$.

η Material constant for couple stress fluid.

λ $\frac{L}{2R}$.

μ Viscosity of the fluid.

Ω Rotational velocity.

ρ Density of the fluid.

θ Circumferential angle.

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