Mechanical Testing and Diagnosis

# A NEW METHOD TO DETERMINE THE MOST EFFICIENT ROTATION ANGLE OF A WIND TURBINE 

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#### Abstract

Collecting of clean wind energy is a very actual problem in our modern world of a global warming. The most popular construction of a wind energy station uses horizontal rotor and its axis must be fixed in some certain position during work process. As better by angle we fix the rotor, as more energy will be collected, so the problem of rotor's rotation angle choice can be considered significant and very actual. In this work is proposed to use the polar coordinate cardioid dependence to approximate the real rose of the wind corresponding to a certain location. Symmetry axis of such cardioid will indicate the most efficient rotor's position. For approximation is used least squares method and automated solving of non-linear algebraic equation. Developed technique allows choosing rotor's rotation angle more reasonable, based on wind statistics of certain locality.


Keywords: Wind energy stations, horizontal rotor, rotor orientation, rotation angle, numerical solution

## 1. INTRODUCTION

During every month, the wind direction changes in a wide range of angles. It changes not only for long periods, but sometimes it happens several times a day. From the practical point of view in Horizontal Axis Wind Turbines (HAWT, [1-2]), it is not very simple process to reset the orientation of a turbine's rotor (connected to the $100-\mathrm{m}$ blades) and it is not possible to carry out it very often (for example, every hour) [3-

4]. Thus, it is expedient to select the most efficient position of a rotor's axis (orientation) for as longer periods as it is possible.

## 2. MATERIALS AND METHODS

The simplest way is to determine the direction of a maximum wind and to orient rotor's axis in this direction (Fig. 1).


Fig. 1. Example of a wind rose with one well-defined wind direction (Odessa, July 2021, values from WeatherArchive.ru [5])


| Direction |  | Percentage |
| :---: | :---: | :---: |
| $\downarrow$ | North | 12.5\% |
| $\swarrow$ | North-East | 25.4\% |
| $\leftarrow$ | East | 8.8\% |
| K | South-East | 0.8\% |
| $\uparrow$ | South | 6.7\% |
| $\nearrow$ | South-West | 7.1\% |
| $\rightarrow$ | West | 11.3\% |
| $\searrow$ | North-West | 27.5\% |

Fig. 2. Example of rose of a wind with hardly defined effective rotor's axis orientation (Odessa, October 2021, values from WeatherArchive.ru)


Fig. 3. Example of a cardioid dependence,

$$
\text { with } \varphi_{0}=30^{\circ} \text { and } A=10
$$

But if rose of winds would be very asymmetric and without one definite maximum, like that in Fig. 2, it is expedient to turn rotor's axis to some mean windward side (to the North - on Fig. 2).

So, we need to use more complicated method to find the most effective direction of a rotor's axis. In this work, we propose to approximate wind distribution via polar coordinates dependence, known as cardioid [6]. The formula is:

$$
\begin{equation*}
\rho(\phi)=A\left(1+\cos \left(\phi+\phi_{0}\right)\right), \tag{1}
\end{equation*}
$$

where parameter $\varphi_{0}$ defines the angle of a graph's rotation (positive values correspond to clockwise rotation) and parameter $A$ defines the size of a graph Fig. 3.

So, the real wind distribution (as in Fig. 1 and Fig. 2, data from http://weatherarchive.ru)) is to be approximated with formula (1), finding current parameter $\varphi_{0}$ and setting rotor's axis correspondingly to this direction.

For the approximation, we have a set of $\varphi_{i}$ values and corresponding $\rho_{i}$ values, presented in tables at the right side of Fig. 1 and Fig. 2. The objective is to find the $\varphi_{0}$ value and it is possible to use Least Square Method (LSM) [7-8]. The objective function is the sum of the squared residuals

$$
\begin{align*}
& \quad \varepsilon_{i}=A\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)-\rho_{i}: \\
& S=\sum_{i} \varepsilon_{i}^{2}=\sum_{i}\left(A\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)-\rho_{i}\right)^{2} \rightarrow \\
& \min \tag{2}
\end{align*}
$$

Traditional minimum condition [9] is:

$$
\frac{\partial S}{\partial A}=0, \quad \frac{\partial S}{\partial \phi_{0}}=0
$$

Differentiating (and after reduction by 2 ), we will get the system of non-linear algebraic equations:

$$
\left\{\begin{array}{l}
\sum_{i}\left(A\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)-\rho_{i}\right)\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)=0 \\
\sum_{i}\left(A\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)-\rho_{i}\right) A\left(-\sin \left(\phi_{i}+\phi_{0}\right)\right)=0
\end{array}\right.
$$

Transforming:

$$
\left\{\begin{array}{c}
\sum_{i} A\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)^{2}-\sum_{i} \rho_{i}\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)=0 \\
\sum_{i} A^{2}\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right) \sin \left(\phi_{i}+\phi_{0}\right)-\sum_{i} A \rho_{i} \sin \left(\phi_{i}+\phi_{0}\right)=0
\end{array}\right.
$$

Expressing $A$ from the second equation and substituting it into the first one, we will get the nonlinear equation of which it is possible to find parameter $\varphi_{0}$ :
$\sum_{i} \rho_{i} \sin \left(\phi_{i}+\phi_{0}\right) \cdot \sum_{i}\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right)^{2}-$
$-\sum_{i} \rho_{i}\left(1+\cos \left(\phi_{i}+\phi_{0}\right)\right) \cdot \sum_{i}\left(1+\cos \left(\phi_{i}+\right.\right.$
$\left.\left.\phi_{0}\right)\right) \sin \left(\phi_{i}+\phi_{0}\right)=0$

## 3. RESULTS

Using some mathematical software (its choice is very wide because solving of non-linear algebraic equations is a standard task for such programs (Fig. 4) [10-12], it is possible to get the root of (3) (i.e., desired angle) for any real rose of winds. For example, in Fig. 5, there are shown two cases for Odessa city, 2021 year, February $\left(\varphi_{0}=192^{\circ}\right)$ and $\operatorname{March}\left(\varphi_{0}=259^{\circ}\right)$.


$$
\varphi:=\left(\begin{array}{c}
0 \\
45 \\
90 \\
135 \\
180 \\
225 \\
270 \\
315
\end{array}\right) \quad \alpha_{i}:=\varphi_{i} \cdot \frac{\pi}{180}
$$

$$
\mathrm{x}:=1
$$

$$
\underset{\text { F }}{ }(\mathrm{x}):=\sum_{i}\left(\rho_{i} \cdot \sin \left(\alpha_{i}+x\right)\right) \cdot \sum_{i}\left(1+\cos \left(\alpha_{i}+x\right)\right)^{2}-\sum_{i}\left[\rho_{i} \cdot\left(1+\cos \left(\alpha_{i}+x\right)\right)\right] \cdot \sum_{i}\left[\left(1+\cos \left(\alpha_{i}+x\right)\right) \cdot \sin \left(\alpha_{i}+x\right)\right]
$$

$$
\alpha_{1}:=\operatorname{root}(\mathrm{F}(\mathrm{x}), \mathrm{x}) \quad \alpha_{1}=-0.148 \quad \alpha_{0}:=\alpha_{1} \cdot \frac{180}{\pi}+180 \quad \alpha_{0}=171.532
$$

$$
\rho 2(\varphi):=a \cdot\left(1+\cos \left(\varphi+\frac{\alpha_{0} \cdot \pi}{180}\right)\right)
$$



Fig. 4. Mathematical software using to solve non-linear algebraic equation

a) b)

Fig. 5. Example of proposed method using for locality of Odessa in 2021 year: a - February, b-March

## 4. DISCUSSION

The difference between two values of rotor's orientation angle (traditional direction to maximum wind and proposed in this work direction of an approximated cardioid axis) may be great enough. For example, in Fig. 5a, cardioid angle is $168^{\circ}\left(\varphi_{0}=192^{\circ}\right.$ if it counts clockwise positive angles from the polar axis), but the maximum wind is observed on the angle of $135^{\circ}$, so the difference is $168^{\circ}-135^{\circ}=33^{\circ}$, that is great enough and it can give a noticeable difference in energy collected amounts.

## CONCLUSION

So, in this work it is proposed the method to find the most efficient wind energy station rotor's orientation for any rose of winds. For this purpose, cardioid approximation with LSM method is used. In further work, it is planned to experimentally estimate the efficiency of traditional rotor's orientation on maximum wind and on the direction provided by the method proposed in this work.

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