# SURFACE PROFILING METHOD OF THE DISK CUTTER OF THE MALE ROTOR FROM THE SCREW COMPRESSOR COMPONENT 

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#### Abstract

The active surfaces of the screw compressors rotors are cylindrical helicoidal surfaces, with constant step; these surfaces are complex surfaces in order to meet a set of specifics conditions.

This paper presents, according to the theorems referring to enveloping surfaces, a specific application regarding the principle of replacement of elementary generators surfaces belonging to the set of helicoidal surfaces described by dot matrix, using the tangent method, in order to determine the shape of disk cutter profile for the male rotor. Also, based on a Java soft product, we present frontal applications of the axial shapes of the generating disk cutter for the male rotor, component of screw compressor, gear ratio 4/6 and 3/5.


KEYWORDS: helicoidal surfaces, Bézier polynomial, meshing surfaces, disk cutter

## 1. Introduction

The screw compressor rotors profiling (in cross section), starts with the rack generating [6], according to the enveloping method [1], [2], in order to determine, analytically or numerical, by these shapes.

Once known the shape in cross section, we can get the transverse profile of the meshing rotor [7], [8], [9], [10] solving the problem as a matter of enveloping with plane profiles belonging to one couple of centrodes.

The enveloping surfaces study can be done based on the fundamental theorems of enveloping surfaces, Olivier and Gohman theorems [1]; we can also use one of complementary theorems: "The minimum distance method" [3] and "The substitutive circles family" [3], or "The method of plane trajectory generation" [4]. The graphics methods 2D or 3D can solve the problem also [5], [6], [13].

The screw compressor active surfaces can be manufactured using the disk cutter, whose primary peripheral surfaces are revolution ones conjugated with rotors helical surfaces.

## 2. The helical surfaces of male rotor

The coordinates system attached to disk cutter and its circular centrode are defined in Figure 1. Table 1
presents the profiles of the generating rack for the system $\xi \eta$.

The cinematic process of generation means the rolling of the two centrodes: one linear C , of the rack, the other one circular, C 1 , radius Rr 1 , specific to cross section of the male rotor.


Fig. 1. Rolling centrodes; male rotor profiling

In Table 2, we present the analytic models of profiles and meshing conditions). transverse section of male rotor (family of plane

Table 1. The rack profiles

| Sg. | Profiles | Variable parameters |
| :---: | :---: | :---: |
| $\overparen{A B}$ | $\left\{\begin{array}{l}\xi(\psi)=\mathrm{R}_{0} \cdot \cos \psi-c_{0} ; \\ \eta(\psi)=-\mathrm{R}_{0} \cdot \sin \psi .\end{array}\right.$ | $\begin{gathered} \psi_{\min }=0 ; \\ \psi_{\max }-\text { constructive } \end{gathered}$ |
| $\overline{\mathrm{BC}}$ | $\left\{\begin{array}{l}\xi(u)=\xi_{B}-u \cdot \cos \beta ; \\ \eta(u)=\eta_{B}-u \cdot \sin \beta .\end{array}\right.$ | $\begin{gathered} \mathrm{u}_{\min }=0 ; \\ \mathrm{u}_{\max }-\text { contructive } \\ \beta=\frac{\pi}{2}-\psi_{\max } \end{gathered}$ |
| $\overparen{C D}$ | $\left\{\begin{array}{l}\xi(v)=-\mathrm{r}_{0} \cdot \cos v+\xi_{0_{1}} ; \\ \eta(v)=+\mathrm{r}_{0} \cdot \sin v+\eta_{0_{1}} .\end{array}\right.$ | $\begin{gathered} v_{\min }=0 ; \\ v_{\max }=\frac{\pi}{2}-\beta \end{gathered}$ |
| $\overparen{\text { EF }}$ | $\left\{\begin{array}{l} \xi\left(v_{1}\right)=-r_{0} \cdot \cos v_{1}+\xi_{0_{2}} ; \\ \eta\left(v_{1}\right)=-r_{0} \cdot \sin v_{1}+\eta_{0_{2}} . \end{array}\right.$ | $\begin{gathered} v_{1 \min }=0 ; \\ v_{1 \max }=\frac{\pi}{2}-\beta_{1} \end{gathered}$ |
| FG | $\left\{\begin{array}{l}\xi\left(u_{1}\right)=+u_{1} \cdot \cos \beta_{1}+\xi_{F} ; \\ \eta\left(u_{1}\right)=-u_{1} \cdot \sin \beta_{1}+\eta_{F} .\end{array}\right.$ | $\begin{gathered} \mathrm{u}_{1 \min }=0 ; \\ \mathrm{u}_{1 \max }=\text { constructive } \end{gathered}$ |
| $\widehat{\mathrm{AH}}$ | $\left\{\begin{array}{l} \mathrm{P}_{\xi \mathrm{AH}}=\lambda_{1}^{2} \mathrm{~A}_{\xi}+2\left(1-\lambda_{1}\right) \lambda_{1} \mathrm{~B}_{\xi}+\left(1-\lambda_{1}\right)^{2} \mathrm{C}_{\xi} \\ \mathrm{P}_{\eta \mathrm{AG}}=\lambda_{1}^{2} \mathrm{~A}_{\eta}+2\left(1-\lambda_{1}\right) \lambda_{1} \mathrm{~B}_{\eta}+\left(1-\lambda_{1}\right)^{2} \mathrm{C}_{\eta} \end{array}\right.$ | $0 \leq \lambda_{1} \leq 1$ |
| $\widehat{\mathrm{GH}}$ | $\left\{\begin{array}{l} \mathrm{P}_{\xi \mathrm{HG}}=\lambda_{2}^{2} \mathrm{D}_{\xi}+2\left(1-\lambda_{2}\right) \lambda_{2} \mathrm{E}_{\xi}+\left(1-\lambda_{2}\right)^{2} \mathrm{~F}_{\xi} ; \\ \mathrm{P}_{\eta \mathrm{HG}}=\lambda_{2}^{2} \mathrm{D}_{\eta}+2\left(1-\lambda_{2}\right) \lambda_{2} \mathrm{E}_{\eta}+\left(1-\lambda_{2}\right)^{2} \mathrm{~F}_{\eta}, \end{array}\right.$ | $0 \leq \lambda_{2} \leq 1$ |

The transverse profiles of male rotor, see Figure 1, can be described, according to the analytic formulas, through the coordinate matrix,

$$
\mathrm{G}=\left\|\begin{array}{ll}
\mathrm{X}_{1_{1}} & \mathrm{Y}_{1_{1}}  \tag{1}\\
\mathrm{X}_{1_{2}} & \mathrm{Y}_{1_{2}} \\
\cdots & \cdots \\
\mathrm{X}_{1_{\mathrm{n}}} & \mathrm{Y}_{1_{\mathrm{n}}}
\end{array}\right\|,
$$

thus, the distance between two successive points $\mathrm{M}_{\mathrm{i}}$, $\mathrm{M}_{\mathrm{i}+1}$, is small enough:
$\mathrm{ds}=\sqrt{\left(\mathrm{X}_{\mathrm{li}+1}-\mathrm{x}_{\mathrm{li}}\right)^{2}+\left(\mathrm{Y}_{\mathrm{li}+1}-\mathrm{Y}_{\mathrm{li}}\right)^{2}} \leq \varepsilon,(2)$ $\varepsilon=1 \cdot 10^{-3} \mathrm{~mm}$.
We define $\beta \mathrm{i}$ the angle of inclination of the
elementary segment, $\operatorname{tg} \beta_{\mathrm{i}}=\frac{\left|\mathrm{Y}_{\mathrm{l}_{\mathrm{i}+1}}-\mathrm{Y}_{1 \mathrm{i}}\right|}{\left|\mathrm{X}_{1 \mathrm{i}+1}-\mathrm{X}_{\mathrm{l}}\right|}$,
see Figure 2.

Table 2. Analytic model of transverse profile of male rotor

| Sg. | Family of profiles | Meshing condition | Variable parameters |
| :---: | :---: | :---: | :---: |
| $\overparen{A B}$ | $\left\{\begin{array}{l} \mathrm{X}_{1}=\mathrm{R}_{0} \cdot \cos \left(\psi-\varphi_{1}\right)+\left(\mathrm{R}_{\mathrm{r}_{1}}-\mathrm{c}_{0}\right) \cdot \cos \varphi_{1}+ \\ +\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1} \cdot \sin \varphi_{1} ; \\ \mathrm{Y}_{1}=-\mathrm{R}_{0} \cdot \sin \left(\psi-\varphi_{1}\right)+\left(\mathrm{R}_{\mathrm{r}_{1}}-\mathrm{c}_{0}\right) \cdot \sin \varphi_{1}- \\ -\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1} \cdot \cos \varphi_{1} ; \end{array}\right.$ | $\varphi_{1}=-\frac{\mathrm{c}_{0}}{\mathrm{R}_{\mathrm{r} 1}} \operatorname{tg} \psi$ | $\begin{gathered} \psi_{\min }=0 ; \\ \psi_{\max }=\text { constr. } \end{gathered}$ |
| $\overline{\mathrm{BC}}$ | $\left\{\begin{array}{l} \mathrm{X}_{1}=\mathrm{u} \cdot \sin \left(\varphi_{1}-\psi_{\max }\right)+\left(\mathrm{R}_{\mathrm{r}_{1}}+\xi_{\mathrm{B}}\right) \cdot \cos \varphi_{1}+ \\ +\left(\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1}+\eta_{\mathrm{B}}\right) \cdot \sin \varphi_{1} ; \\ \mathrm{Y}_{1}=-\mathrm{u} \cdot \cos \left(\varphi_{1}-\psi_{\max }\right)-\left(\mathrm{R}_{\mathrm{r}_{1}}+\xi_{\mathrm{B}}\right) \cdot \sin \varphi_{1}+ \\ +\left(\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1}+\eta_{\mathrm{B}}\right) \cdot \cos \varphi_{1} ; \end{array}\right.$ | $\varphi_{1}=\frac{-\frac{u}{\cos \psi_{\max }}+\xi_{\mathrm{B}} \cdot \operatorname{tg} \psi_{\max }-\eta_{\mathrm{B}}}{R_{\mathrm{r}_{1}}}$ | $\begin{gathered} \mathrm{u}_{\min }=0 ; \\ \mathrm{u}_{\max }=\text { constr. } \\ \beta=\frac{\pi}{2}-\psi_{\max } \end{gathered}$ |
| $\overparen{C D}$ | $\left\{\begin{array}{l} X_{1}=-r_{0} \cdot \cos \left(\varphi_{1}-v\right)+\left(R_{r_{1}}+\xi_{0_{1}}\right) \cdot \cos \varphi_{1}- \\ -\left(-R_{r_{1}} \cdot \varphi_{1}+\eta_{0_{1}}\right) \cdot \sin \varphi_{1} ; \\ Y_{1}=-r_{0} \cdot \sin \left(\varphi_{1}-v\right)+\left(R_{r_{1}}+\xi_{0_{1}}\right) \cdot \sin \varphi_{1}+ \\ +\left(-R_{r 1} \cdot \varphi_{1}+\eta_{0_{1}}\right) \cdot \cos \varphi_{1} ; \end{array}\right.$ | $\varphi_{1}=\frac{\xi_{0_{1}} \cdot \operatorname{tg} v+\eta_{0_{1}}}{\mathrm{R}_{\mathrm{r}_{1}}}$ | $\begin{gathered} v_{\min }=0 ; \\ v_{\max }=\frac{\pi}{2}-\beta \end{gathered}$ |
| $\overparen{\text { EF }}$ | $\left\{\begin{array}{l} X_{1}=-r_{0} \cdot \cos \left(v_{1}+\varphi_{1}\right)+\left(\mathrm{R}_{\mathrm{r}_{1}}+\xi_{0_{2}}\right) \cdot \cos \varphi_{1}+ \\ +\left(\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1}-\eta_{0_{2}}\right) \cdot \sin \varphi_{1} \\ \mathrm{Y}_{1}=-\mathrm{r}_{0} \cdot \sin \left(v_{1}+\varphi_{1}\right)-\left(\mathrm{R}_{\mathrm{r}_{1}}+\xi_{0_{2}}\right) \cdot \sin \varphi_{1}+ \\ +\left(\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1}+\eta_{0_{2}}\right) \cdot \cos \varphi_{1} \end{array}\right.$ | $\varphi_{1}=\frac{\xi_{0_{2}} \cdot \operatorname{tg} v_{1}+\eta_{0_{2}}}{\mathrm{R}_{\mathrm{r}_{1}}}$ | $\begin{gathered} v_{1 \min }=0 ; \\ v_{1 \max }=\frac{\pi}{2}-\beta_{1} \end{gathered}$ |
| $\overline{\mathrm{FG}}$ | $\left\{\begin{array}{l} X_{1}=u_{1} \cdot \cos \left(\varphi_{1}-\beta_{1}\right)+\left(R_{\mathrm{r}_{1}}+\xi_{\mathrm{F}}\right) \cdot \cos \varphi_{1}- \\ -\left(-\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1}+\eta_{\mathrm{F}}\right) \cdot \sin \varphi_{1} ; \\ \mathrm{Y}_{1}=\mathrm{u}_{1} \cdot \sin \left(\varphi_{1}-\beta_{1}\right)+\left(\mathrm{R}_{\mathrm{r}_{1}}+\xi_{\mathrm{F}}\right) \cdot \sin \varphi_{1}+ \\ +\left(-\mathrm{R}_{\mathrm{r}_{1}} \cdot \varphi_{1}+\eta_{\mathrm{F}}\right) \cdot \cos \varphi_{1} ; \end{array}\right.$ | $\varphi_{1}=\frac{-\frac{\mathrm{u}_{1}}{\sin \beta_{1}}+\xi_{\mathrm{F}} \cdot \operatorname{ctg} \beta_{1}+\eta_{\mathrm{F}}}{\mathrm{R}_{\mathrm{r}_{1}}}$ | $\begin{gathered} \mathrm{u}_{1 \min }=0 ; \\ \mathrm{u}_{1 \max }=\text { constr. } . \end{gathered}$ |
| $\widehat{\text { AH }}$ | $\left\{\begin{array}{l} X_{1}=\left(\xi(\lambda)+\mathrm{R}_{\mathrm{r} 1}\right) \cdot \cos \varphi_{1}- \\ -\left(\eta(\lambda)-\mathrm{R}_{\mathrm{r} 1} \cdot \varphi_{1}\right) \cdot \sin \varphi_{1} ; \\ \mathrm{Y}_{1}=-\left(\xi(\lambda)-\mathrm{R}_{\mathrm{r} 1}\right) \cdot \sin \varphi_{1}+ \\ +\left(\eta(\lambda)-\mathrm{R}_{\mathrm{r} 1} \cdot \varphi_{1}\right) \cdot \cos \varphi_{1} ; \end{array}\right.$ | $\frac{\dot{\mathrm{X}}_{1_{\lambda_{1}}}}{\dot{\mathrm{X}}_{1_{\varphi_{1}}}}=\frac{\dot{\mathrm{Y}}_{1_{\lambda_{1}}}}{\dot{\mathrm{Y}}_{1_{\varphi_{1}}}}$ | $\lambda_{1}, \lambda_{2}$ |
| $\widehat{\text { GH }}$ | $\left\{\begin{array}{l} \mathrm{X}_{1}=\left(\xi(\lambda)+\mathrm{R}_{\mathrm{r} 1}\right) \cdot \cos \varphi_{1}- \\ -\left(\eta(\lambda)-\mathrm{R}_{\mathrm{r} 1} \cdot \varphi_{1}\right) \cdot \sin \varphi_{1} \\ \mathrm{Y}_{1}=-\left(\xi(\lambda)-\mathrm{R}_{\mathrm{r} 1}\right) \cdot \sin \varphi_{1}+ \\ +\left(\eta(\lambda)-\mathrm{R}_{\mathrm{r} 1} \cdot \varphi_{1}\right) \cdot \cos \varphi_{1} \end{array}\right.$ | $\frac{\dot{\mathrm{X}}_{1_{\lambda_{2}}}}{\dot{\mathrm{X}}_{1_{\varphi_{1}}}}=\frac{\dot{\mathrm{Y}}_{1_{\lambda_{2}}}}{\dot{\mathrm{Y}}_{\varphi_{\varphi_{1}}}}$ | $\lambda_{1}, \lambda_{2}$ |



Fig. 2. The helicoidal surfaces of male rotor; coordinates system

To express the segment $\mathrm{M}_{\mathrm{i}} \mathrm{M}_{\mathrm{i}+1}$ :

$$
M_{i} M_{i+1} \left\lvert\, \begin{align*}
& X_{1}=X_{1 i}+\lambda \cdot \cos \beta_{i}  \tag{4}\\
& Y_{1}=Y_{1 i}+\lambda \cdot \sin \beta_{i}
\end{align*}\right.
$$

with $\lambda_{\text {min }}=0 ; \lambda_{\max }=\mathrm{d}_{\mathrm{S}}$, see (2).
We imagine an elementary helicoidal surface, described by the coordinates transformation as follows:

$$
\left\|\begin{array}{l}
\mathrm{X}_{1}  \tag{5}\\
\mathrm{Y}_{1} \\
\mathrm{Z}_{1}
\end{array}\right\|=\omega_{3}^{\mathrm{T}}\left(\theta_{1}\right) \cdot\left\|\begin{array}{l}
\mathrm{X}_{\mathrm{l}_{\mathrm{i}}} \\
\mathrm{Y}_{\mathrm{i}} \\
0
\end{array}\right\|+\left\|\begin{array}{l}
0 \\
0 \\
\mathrm{p}_{1} \cdot \theta_{1}
\end{array}\right\|
$$

right helix, the helicoidal parameter $\mathrm{p}_{1}$ and $\theta_{1}$ the variable parameter.

The surfaces assembly (5) ( $\mathrm{i}=1 \ldots \mathrm{n}$ ), represents an accurate substitution of the male rotor flank, corresponding to $\mathrm{AB}, \mathrm{BC}$, etc, see Table 2.

Obviously, the normal to the helicoidal elementary surface (5) can be written as follows:

$$
\begin{equation*}
\overrightarrow{\mathrm{N}}_{\Sigma}=\overrightarrow{\mathrm{T}}_{\mathrm{M}_{\mathrm{i}} \mathrm{M}_{\mathrm{i}+1}} \times \overrightarrow{\mathrm{T}}_{\mathrm{M}_{\mathrm{i}}} \tag{6}
\end{equation*}
$$

where

$$
\begin{gather*}
\overrightarrow{\mathrm{T}}_{\mathrm{M}_{\mathrm{i}}}=\frac{\mathrm{dX}}{1} \mathrm{~d} \theta_{1} \cdot \overrightarrow{\mathrm{i}}+\frac{\mathrm{dY}}{1} \frac{\mathrm{~d} \theta_{1}}{\mathrm{j}}+\mathrm{p}_{1} \cdot \overrightarrow{\mathrm{k}}  \tag{7}\\
\overrightarrow{\mathrm{~T}}_{\mathrm{M}_{\mathrm{i}} \mathrm{M}_{\mathrm{i}+1}}=\cos \beta_{\mathrm{i}} \cdot \overrightarrow{\mathrm{i}}+\sin \beta_{\mathrm{i}} \cdot \overrightarrow{\mathrm{j}} \tag{8}
\end{gather*}
$$

The normal to the elementary helicoidal surface is calculated as follows:

$$
\overrightarrow{\mathrm{N}}_{\Sigma}=\left|\begin{array}{ccc}
\overrightarrow{\mathrm{i}} & \overrightarrow{\mathrm{j}} & \overrightarrow{\mathrm{k}}  \tag{9}\\
-\mathrm{X}_{1 \mathrm{i}} \sin \theta_{1}-\mathrm{Y}_{1 \mathrm{i}} \cos \theta_{1} & \mathrm{X}_{1 \mathrm{i}} \cos \theta_{1}-\mathrm{Y}_{1 \mathrm{i}} \sin \theta_{1} & \mathrm{p}_{1} \\
\cos \beta_{\mathrm{i}} & \sin \beta_{\mathrm{i}} & 0
\end{array}\right|,(\mathrm{C}
$$

or:

$$
\begin{equation*}
\overrightarrow{\mathrm{N}}_{\Sigma}=\mathrm{N}_{\mathrm{X}_{1}} \cdot \overrightarrow{\mathrm{i}}+\mathrm{N}_{\mathrm{Y}_{1}} \cdot \overrightarrow{\mathrm{j}}+\mathrm{N}_{\mathrm{Z}_{1}} \cdot \overrightarrow{\mathrm{k}} \tag{10}
\end{equation*}
$$

by the definitions:

$$
\left\{\begin{array}{l}
\mathrm{N}_{\mathrm{X}_{1}}=-\mathrm{p}_{1} \cdot \sin \beta_{\mathrm{i}} ; \\
\mathrm{N}_{\mathrm{Y}_{2}}=\mathrm{p}_{1} \cdot \cos \beta_{\mathrm{i}} ; \\
\mathrm{N}_{\mathrm{Z}_{2}}=\sin \beta_{\mathrm{i}} \cdot\left[-\mathrm{X}_{1_{\mathrm{i}}} \cdot \sin \theta_{1}-\mathrm{Y}_{1_{\mathrm{i}}} \cdot \cos \theta_{1}\right]-  \tag{11}\\
-\cos \beta_{\mathrm{i}} \cdot\left[\mathrm{X}_{1_{\mathrm{i}}} \cdot \cos \theta_{1}-\mathrm{Y}_{1_{\mathrm{i}}} \cdot \sin \theta_{1}\right]
\end{array}\right.
$$

## 3. Profiling the disk cutter - algorithm

The axis position of disk cutter, see Figure 2, is defined:

$$
\begin{equation*}
\overrightarrow{\mathrm{A}}=-\sin \alpha \cdot \overrightarrow{\mathrm{j}}+\cos \alpha \cdot \overrightarrow{\mathrm{k}} \tag{12}
\end{equation*}
$$

and the amount of the angle $\alpha$,

$$
\begin{equation*}
\operatorname{tg} \alpha=\frac{2 \pi \cdot \mathrm{p}_{1}}{\pi \cdot \mathrm{D}_{\mathrm{e}_{1}}}=\frac{2 \cdot \mathrm{p}_{1}}{\mathrm{D}_{\mathrm{e}_{1}}} \tag{13}
\end{equation*}
$$

where $\mathrm{p}_{1}$ is the helicoidally parameter and $\mathrm{De}_{1}$ represents the outlet diameter of male rotor.

Thus, the condition of characteristic curve determination on the elementary helicoidally surfaces becomes

$$
\begin{equation*}
\left(\overrightarrow{\mathrm{N}}_{\Sigma}, \overrightarrow{\mathrm{A}}, \overrightarrow{\mathrm{r}}_{2}\right)=0 \tag{14}
\end{equation*}
$$

where:

$$
\begin{align*}
& \overrightarrow{\mathrm{r}}_{1}=\left[\mathrm{X}_{1 \mathrm{i}} \cdot \cos \theta_{1}-\mathrm{Y}_{1 \mathrm{i}} \cdot \sin \theta_{1}-\mathrm{a}\right] \cdot \overrightarrow{\mathrm{i}}+  \tag{15}\\
& +\left[\mathrm{X}_{1 \mathrm{i}} \cdot \sin \theta_{1}+\mathrm{Y}_{1 \mathrm{i}} \cdot \cos \theta_{1}\right] \cdot \overrightarrow{\mathrm{j}}+\mathrm{p}_{1} \cdot \theta_{1} \cdot \overrightarrow{\mathrm{k}}
\end{align*}
$$

$-\mathrm{X}_{1}, \mathrm{Y}_{1}$ defined by (5);
a - the amount of the inlet radius of male rotor and outlet radius of disk cutter, convenient to be technologically determined.

The envelope condition becomes:

$$
(\overrightarrow{\mathrm{N}}, \overrightarrow{\mathrm{~A}}, \overrightarrow{\mathrm{r}})=\left|\begin{array}{ccc}
\left(\mathrm{X}_{1 \mathrm{i}} \cdot \cos \theta_{1}-\mathrm{Y}_{1 \mathrm{l}} \cdot \sin \theta_{1}-\mathrm{a}\right) & \left(\mathrm{X}_{1 \mathrm{i}} \cdot \sin \theta_{1}+\mathrm{Y}_{1 \mathrm{i}} \cdot \cos \theta_{1}\right) & +\mathrm{p}_{1} \cdot \theta_{1}  \tag{16}\\
-\mathrm{p}_{1} \cdot \sin \beta_{\mathrm{i}} & \mathrm{p}_{1} \cdot \cos \beta_{\mathrm{i}} & \left(\mathrm{X}_{1 \mathrm{i}} \cdot \cos \left(\theta_{1}-\beta_{\mathrm{i}}\right)+\mathrm{Y}_{1 \mathrm{i}} \cdot \sin \left(\theta_{1}-\beta_{\mathrm{i}}\right)\right) \\
-\sin \alpha & \cos \alpha & 0
\end{array}\right| \leq \varepsilon
$$

surface of disk cutter. It can be expressed as the vector,

$$
X_{1}^{\mathrm{c}}=\left\{\begin{array}{lll}
\mathrm{X}_{1_{\mathrm{i}}}^{\mathrm{c}} & \mathrm{Y}_{1_{\mathrm{i}}}^{\mathrm{c}} & \mathrm{Z}_{1_{i}}^{\mathrm{c}} \tag{17}
\end{array}\right\}^{\mathrm{T}}, \quad(\mathrm{i}=1 \ldots \mathrm{~m})
$$

Changing the coordinates system:

$$
\begin{align*}
& X_{1 S}=\alpha \cdot\left(X_{1}-\mathrm{a}\right) ; \\
& \alpha=\left\|\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{array}\right\| ; a=\left\|\begin{array}{l}
a \\
0 \\
0
\end{array}\right\|, \tag{18}
\end{align*}
$$

is obtained the expression of the characteristic curve in the system of disk cutter $\mathrm{X}_{1 \mathrm{~S}} \mathrm{Y}_{1 \mathrm{~S}} \mathrm{Z}_{1 \mathrm{~S}}$, see Figure 3:

$$
\begin{aligned}
& \left\{\begin{array}{l}
X_{1 S}=X_{1 i}^{c}-\mathrm{a} \\
\mathrm{Y}_{1 \mathrm{~S}}=\mathrm{Y}_{\mathrm{li}}^{\mathrm{c}} \cdot \cos \alpha+\mathrm{Z}_{\mathrm{li}}^{\mathrm{c}} \cdot \sin \alpha \\
\mathrm{Z}_{1 \mathrm{~S}}=-\mathrm{Y}_{\mathrm{li}}^{\mathrm{c}} \cdot \sin \alpha+\mathrm{Z}_{\mathrm{li}}^{\mathrm{c}} \cdot \cos \alpha . \\
\mathrm{i}=(1 \ldots . \ldots \mathrm{m})
\end{array}\right.
\end{aligned}
$$

The axial section of the disk cutter is obtained:

$$
\begin{align*}
& \left\{\begin{array}{l}
\mathrm{H}=-\mathrm{Y}_{\mathrm{li}}^{\mathrm{C}} \cdot \sin \alpha+\mathrm{Z}_{\mathrm{li}}^{\mathrm{C}} \cdot \cos \alpha ; \\
\mathrm{R}=\sqrt{\left[\mathrm{X}_{\mathrm{li}}^{\mathrm{C}}-\mathrm{a}\right]^{2}+\left[\mathrm{Y}_{\mathrm{li}}^{\mathrm{C}} \cdot \cos \alpha+\mathrm{Z}_{\mathrm{li}}^{\mathrm{C}} \cdot \sin \alpha\right]^{2}}
\end{array}\right.  \tag{20}\\
& \mathrm{i}=(1 \ldots \mathrm{~m}) .
\end{align*}
$$

According to this algorithm, all the surface constituents of male rotor corresponding to Table 2, can be solved.

## 4. Numerical examples

We present two frontal applications, different constructive solutions.

In Table 4 and Figure 3 are described the axial section coordinates of disk cutter for the male rotor.


Fig. 3. Male rotor - the tooth profile of disk cutter


Fig. 4. The solid of disk cutter for male rotor

## First application (screw compressor, ratio 4/6)

Table 3. The constructive data of the rack generating, (see Figure 2)

| $\mathrm{R}_{0}[\mathrm{~mm}]$ | $\mathrm{r}_{0}[\mathrm{~mm}]$ | $\mathrm{u}_{\max }[\mathrm{mm}]$ | $\psi_{\max }\left[{ }^{0}\right]$ | $v_{\max }\left[{ }^{0}\right]$ | $v_{1 \max }$ <br> $\left[{ }^{0}\right]$ | $\mathrm{u}_{1 \max }[\mathrm{~mm}]$ | $\mathrm{L}_{\mathrm{p}}[\mathrm{mm}]$ | $\mathrm{c}_{0}[\mathrm{~mm}]$ | $\mathrm{Rr}_{2}[\mathrm{~mm}]$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 22.000 | 1.100 | 10.300 | 63.400 | 63.400 | 58.285 | 6.451 | 50.265 | 4.000 | 32.000 |

Table 4. The axial profile of disk cutter coordinates of male rotor

| Nr. crt. | R [mm] | $\mathbf{H}[\mathbf{m m}]$ | Nr. crt. | $\mathbf{R}[\mathbf{m m}]$ | $\mathbf{H}[\mathbf{m m}]$ | Nr. crt. | $\mathbf{R}[\mathbf{m m}]$ | $\mathbf{H}[\mathbf{m m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 31.398 | -13.952 | $\mathbf{4 1}$ | 47.071 | -1.449 | $\mathbf{8 1}$ | 37.437 | 5.413 |
| $\mathbf{2}$ | 31.366 | -13.408 | $\mathbf{4 2}$ | 47.598 | -1.306 | $\mathbf{8 2}$ | 36.974 | 5.701 |
| $\mathbf{3}$ | 31.369 | -12.863 | $\mathbf{4 3}$ | 48.126 | -1.171 | $\mathbf{8 3}$ | 36.516 | 5.997 |
| $\mathbf{4}$ | 31.408 | -12.320 | $\mathbf{4 4}$ | 48.654 | -1.036 | $\mathbf{8 4}$ | 36.069 | 6.309 |
| $\mathbf{5}$ | 31.483 | -11.780 | $\mathbf{4 5}$ | 49.185 | -0.914 | $\mathbf{8 5}$ | 35.629 | 6.630 |
| $\mathbf{6}$ | 31.595 | -11.247 | $\mathbf{4 6}$ | 49.717 | -0.793 | $\mathbf{8 6}$ | 35.199 | 6.966 |
| $\mathbf{7}$ | 31.751 | -10.725 | $\mathbf{4 7}$ | 50.248 | -0.670 | $\mathbf{8 7}$ | 34.781 | 7.315 |
| $\mathbf{8}$ | 31.952 | -10.218 | $\mathbf{4 8}$ | 50.777 | -0.536 | $\mathbf{8 8}$ | 34.373 | 7.677 |
| $\mathbf{9}$ | 32.186 | -9.727 | $\mathbf{4 9}$ | 51.305 | -0.402 | $\mathbf{8 9}$ | 33.982 | 8.057 |
| $\mathbf{1 0}$ | 32.449 | -9.249 | $\mathbf{5 0}$ | 51.805 | -0.224 | $\mathbf{9 0}$ | 33.600 | 8.445 |
| $\mathbf{1 1}$ | 32.761 | -8.802 | $\mathbf{5 1}$ | 52.149 | 0.199 | $\mathbf{9 1}$ | 33.247 | 8.861 |
| $\mathbf{1 2}$ | 33.091 | -8.369 | $\mathbf{5 2}$ | 52.279 | 0.604 | $\mathbf{9 2}$ | 32.906 | 9.286 |
| $\mathbf{1 3}$ | 33.448 | -7.957 | $\mathbf{5 3}$ | 51.868 | 0.962 | $\mathbf{9 3}$ | 32.589 | 9.729 |


| $\mathbf{1 4}$ | 33.831 | -7.570 | $\mathbf{5 4}$ | 51.427 | 1.258 | $\mathbf{9 4}$ | 32.305 | 10.194 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5}$ | 34.226 | -7.194 | $\mathbf{5 5}$ | 50.889 | 1.342 | $\mathbf{9 5}$ | 32.045 | 10.673 |
| $\mathbf{1 6}$ | 34.648 | -6.849 | $\mathbf{5 6}$ | 50.350 | 1.426 | $\mathbf{9 6}$ | 31.819 | 11.169 |
| $\mathbf{1 7}$ | 35.074 | -6.509 | $\mathbf{5 7}$ | 49.811 | 1.508 | $\mathbf{9 7}$ | 31.642 | 11.684 |
| $\mathbf{1 8}$ | 35.521 | -6.197 | $\mathbf{5 8}$ | 49.269 | 1.563 | $\mathbf{9 8}$ | 31.506 | 12.211 |
| $\mathbf{1 9}$ | 35.972 | -5.890 | $\mathbf{5 9}$ | 48.726 | 1.617 | $\mathbf{9 9}$ | 31.413 | 12.748 |
| $\mathbf{2 0}$ | 36.436 | -5.606 | $\mathbf{6 0}$ | 48.186 | 1.693 | $\mathbf{1 0 0}$ | 31.370 | 13.291 |

The flank of male rotor is a helicoidal cylindrical surface, constant step, right helix, helicoidal parameter $\mathrm{p}_{1}$ and $\mathrm{a}=\mathrm{R}_{\mathrm{s}}+\mathrm{R}_{1 \text { int }}$.
Table 5. The geometrical constructive elements of the male rotor

| $\mathrm{Rr}_{1}$ <br> $[\mathrm{~mm}]$ | $\mathrm{Rext}_{1}$ <br> $[\mathrm{~mm}]$ | a <br> $[\mathrm{mm}]$ | $\mathrm{p}_{1}$ <br> $[\mathrm{~mm}]$ | $\mathrm{R}_{\mathrm{S}}$ <br> $[\mathrm{mm}]$ | $\beta\left[^{\circ}\right]$ |
| :---: | :--- | :--- | :--- | :--- | :--- |
| 32.0 | 53.0 | 100.0 | 19.099 | 68 | 57.205 |

Helical parameters are calculated with equation:

$$
\begin{equation*}
\mathrm{p}_{2}=\left(\frac{360^{\circ}}{300^{\circ} \cdot \mathrm{i}} \cdot \mathrm{D}_{1}\right) \cdot \frac{1}{2 \cdot \pi} \tag{21}
\end{equation*}
$$

with $\mathrm{i}=4 / 6$ or $3 / 5$.


Fig. 5. Solid model and the geometrical constructive elements of male rotor

## Second application (screw compressor, ratio 3/5)

Table 6. The constructive data of the reference rack, (see Figure 2)

| $\mathrm{R}_{0}[\mathrm{~mm}]$ | $\mathrm{r}_{0}[\mathrm{~mm}]$ | $\mathrm{u}_{\max }[\mathrm{mm}]$ | $\psi_{\max }\left[{ }^{0}\right]$ | $v_{\max }\left[{ }^{0}\right]$ | $v_{\max }$ <br> $\left[{ }^{0}\right]$ | $\mathrm{u}_{1 \max }[\mathrm{~mm}]$ | $\mathrm{L}_{\mathrm{p}}[\mathrm{mm}]$ | $\mathrm{c}_{0}[\mathrm{~mm}]$ | $\mathrm{Rr}_{2}[\mathrm{~mm}]$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 22.000 | 2.000 | 7.045 | 70.300 | 70.300 | 35.054 | 7.774 | 62.832 | 4.000 | 50.000 |

Table 7. The axial profile of disk cutter coordinates of male rotor

| Nr. crt. | R [mm] | $\mathbf{H}[\mathbf{m m}]$ | Nr. crt. | R $[\mathbf{m m}]$ | H [mm] | Nr. crt. | R [mm] | H [mm] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 52.708 | -19.046 | $\mathbf{4 1}$ | 68.092 | -1.669 | $\mathbf{8 1}$ | 59.354 | 9.202 |
| $\mathbf{2}$ | 52.660 | -18.427 | $\mathbf{4 2}$ | 68.611 | -1.326 | $\mathbf{8 2}$ | 58.829 | 9.533 |
| $\mathbf{3}$ | 52.637 | -17.806 | $\mathbf{4 3}$ | 69.121 | -0.972 | $\mathbf{8 3}$ | 58.313 | 9.878 |
| $\mathbf{4}$ | 52.645 | -17.185 | $\mathbf{4 4}$ | 69.618 | -0.600 | $\mathbf{8 4}$ | 57.806 | 10.237 |
| $\mathbf{5}$ | 52.683 | -16.566 | $\mathbf{4 5}$ | 70.105 | -0.215 | $\mathbf{8 5}$ | 57.311 | 10.611 |
| $\mathbf{6}$ | 52.754 | -15.949 | $\mathbf{4 6}$ | 70.594 | 0.168 | $\mathbf{8 6}$ | 56.827 | 11.001 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $\mathbf{1 6}$ | 55.311 | -10.385 | $\mathbf{5 6}$ | 73.980 | 4.334 | $\mathbf{9 6}$ | 53.107 | 15.886 |
| $\mathbf{1 7}$ | 55.727 | -9.924 | $\mathbf{5 7}$ | 73.428 | 4.615 | $\mathbf{9 7}$ | 52.917 | 16.477 |
| $\mathbf{1 8}$ | 56.163 | -9.483 | $\mathbf{5 8}$ | 72.832 | 4.787 | $\mathbf{9 8}$ | 52.777 | 17.081 |
| $\mathbf{1 9}$ | 56.619 | -9.061 | $\mathbf{5 9}$ | 72.220 | 4.888 | $\mathbf{9 9}$ | 52.692 | 17.696 |
| $\mathbf{2 0}$ | 57.091 | -8.657 | $\mathbf{6 0}$ | 71.605 | 4.978 | $\mathbf{1 0 0}$ | 52.664 | 18.316 |



Fig. 6. Male rotor - the tooth profile of disk cutter
In Table 7 and Figure 6, are described the axial section coordinates of disk cutter for the male rotor.


Fig. 8. The solid of disk cutter for male rotor
The flank of male rotor helicoidal surface is a helicoidal cylindrical surface, constant steep, right helix, helicoidal parameter $\mathrm{p}_{1}$ and $\mathrm{a}=\mathrm{R}_{\mathrm{s}}+\mathrm{R}_{1 \text { int }}$.
Table 5. The geometrical constructive elements of male rotor

| $\mathrm{Rr}_{1}$ <br> $[\mathrm{~mm}]$ | $\mathrm{Rext}_{1}$ <br> $[\mathrm{~mm}]$ | a <br> $[\mathrm{mm}]$ | $\mathrm{p}_{1}$ <br> $[\mathrm{~mm}]$ | $\mathrm{R}_{\mathrm{S}}$ <br> $[\mathrm{mm}]$ | $\beta\left[{ }^{\circ}\right]$ |
| :---: | :--- | :--- | :--- | :--- | :--- |
| 30.0 | 52.0 | 100.0 | 20.245 | 70.0 | 57.318 |

## 5. Product software for determination of disk cutter profile

The product soft was elaborated with Sun Java Development Kit, according to the possibility of transverse profile of helicoidal surfaces to be approximated by Bézier polynomial superior degree.

The product allows generating the helicoidal surfaces of screw rotors; using the specific enveloping condition, see Table 1 and relation (20), it is possible to calculate the axial section of disk cutter, mutually enveloping the helicoidal surfaces, constant step, representing the grove between two successive lobes, male and female. The tool relative position was defined taking into account the present algorithm.

The flank corresponding to rotors generator, $A B, B C, C D$, etc. (see Figure 2), can be approximated
by superior degree polynomial, thus the shape representation is very accurate.

This approach fits perfectly into the object Oriented Programming (00P) paradigm [11], [12].

Application description
In this section, application user interface will be described. The most important visual elements of the application are presented in Figure 9, as follows:

1 - select the type of helix generator profile;
2 - configure various parameters of the generating profile (in the case of "Measured points" a list of measured coordinates should be inserted);

3 - select the tool type;
4 - configure a series of helix parameters: outer diameter, inner diameter and the pitch;

5 - update the helical surface displayed on the screen according to parameters and options selected above;

6 - characteristic curve on helical surface;
7 - the helical surface;
8 - helical surface origin and coordinates system;

9 - tool's axis and origin of coordinates system.


Fig. 9. Application user interface


Fig. 10. Male rotor disk cutter profile

In Figure 10, it is presented the applet for the profiling of the disk cutter which generates the male rotor, the solid model of the worm and the characteristic curve onto its flanks.

In the applet, the significance of the coordinate axis $X$ and $Y$ corresponding to the equation (20):

$$
\begin{equation*}
X \equiv \mathrm{R} ; \quad \mathrm{Y} \equiv \mathrm{H} \tag{22}
\end{equation*}
$$

## 5. Conclusions

This paper proposes a profiling method for the male rotor, setting the shape definition of the rack generating.

Using „The tangent method", the helicoidal surfaces of male rotors were substituted by elementary helicoidal surfaces, in order to decrease the analytic computation.

The numerical example of disk cutter profiling was presented, joined with a 3D solid model of primary peripheral surfaces of disk cutter, for diverse construction versions.

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## Metodă pentru profilarea sculei disc destinate prelucrării rotoarelor din componența compresoarelor elicoidale; produs soft de profilare

## —Rezumat—

Suprafețele active ale compresoarelor cu şurub sunt suprafețe elicoidale cilindrice cu pas constant. Aceste suprafețe sunt suprafețe complexe deoarece trebuie să îndeplinească o serie de condiții specifice.

În această lucrare se prezintă, în concordanță cu teoremele înfăşurării suprafețelor, o aplicație practică privind principiul înlocuirii generatoarelor elementare ale suprafețelor aparținând unui cuplu de suprafețe elicoidale descrise în mod discret, utilizând metoda tangentelor, în scopul de a determina forma frezei disc pentru generarea şurubului conducător.

În lucrare se prezintă şi un soft dedicat pentru profilarea acestor scule, dezvoltat în limbajul de programare Java.

