

## THE THERMAL ANALYSIS OF THE ELECTROEROSION PROCESS

Maria Neagu<sup>1</sup>

<sup>1</sup>”Dunărea de Jos” University of Galați, Manufacturing Engineering Department,  
 Galați, România

### ABSTRACT

*This paper presents an analysis of the electroerosion process. It takes into consideration the temperature field of the electrode for both the cylindrical and the rectangular prism cases. The heat dispersed into the electrode and the loss of power during the electroerosion process are analyzed using experimental measurements and mathematical modeling.*

**Keywords:** machining, materials processing, experiment, electroerosion

### 1. INTRODUCTION

The electroerosion process is one of the most important non-conventional materials processes. It is aimed to manufacture parts that have a complicated shape or that are difficult to process due to their material properties. For these cases, the electroerosion process, with a cylindrical, prismatic or other specially configured shape electrodes, is a process that is used successfully.

In this context, the analysis of the temperature field gives us the opportunity to measure the power that is lost through the electrode and to design a better working regimes with a small tools wear and, consequently, with a higher efficiency.

The measurement of the temperature field of the electrode is realised using a laser-based instrument at the exterior surface of the electrode, at the lower and upper limit,  $T_{jos}$  and  $T_{sus}$ , respectively. The temperature of the cooling fluid (oil) is measured

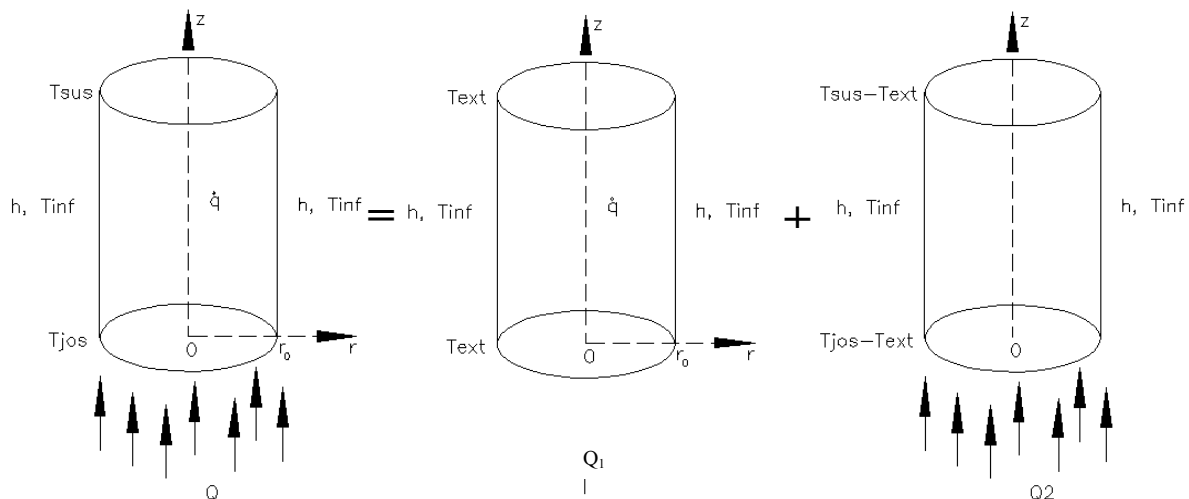
similarly and it is encountered throughout the paper as  $T_{inf}$ .

The author takes into consideration only two cases: the cylindrical and the prismatic electrode. The cylindrical electrode temperature field is modeled in section 2.1, while the prismatic electrode temperature field is modeled in section 2.2, while the results of the experiments will be analyzed by the sections 3.1 and 3.2, respectively.

### 2. MATHEMATICAL MODEL

#### 2.1 Cylindrical electrode

Figure 1 presents the cylindrical electrode of radius  $r_0$  and length  $L$  that is cooled through a natural convection process that is characterized by a convective heat transfer coefficient,  $h$ , and the fluid temperature,  $T_{inf}$ .



**Fig. 1.** The temperature field in a cylindrical electrode during the electroerosion process

The temperatures measured at the lower and the upper limit of the cylinder,  $T_{jos}$  and  $T_{sus}$ , respectively, are due to the two thermal loads that act on the electrode: the electricity and the electrical discharged. The electricity current, through the Joule effect, generates a uniform volumetric thermal load:  $\dot{q}$  [ $W/m^3$ ].

We are separating the effects of these two loads as fig. 1 suggests: a part,  $T_{ext}$  is the temperature due to the Joule effect, while the rest is due to the electric discharged:

$$\theta = \theta_1 + \theta_2. \quad (1)$$

As a consequence, we encounter two well known problems:

1. The temperature of a cylinder thermally loaded with an uniform volumetric heat,  $\dot{q}$ ,  $T_1$ :

$$T_1 = -\frac{\dot{q}r^2}{4k} + \frac{\dot{q}r_0^2}{4k} + \frac{\dot{q}r_0}{2h} + T_\infty; \quad (2)$$

$$\theta_1 = T_1 - T_\infty = -\frac{\dot{q}r^2}{4k} + \frac{\dot{q}r_0^2}{4k} + \frac{\dot{q}r_0}{2h}. \quad (3)$$

Because we measure the temperature at the exterior radius,  $T_{ext}$ , and we consider "x" as the unknown fraction of the temperature increase due to the Joule effect, we can write Eq. (4):

$$\theta_{1,ext} = \theta_1 \Big|_{r=r_0} = \frac{\dot{q}r_0}{2h} = x \cdot \theta_{sus}. \quad (4)$$

But,  $\dot{q}$  is due to the heat load,  $Q_1$ , of the electrode:

$$\dot{q} = \frac{Q_1}{V}. \quad (5)$$

, where V is the volume of the electrode,  $V = \pi r_0^2 L$ .

2. The temperature  $T_2$  is the temperature due to the electrical discharge. The results of fins analysis can be used:

$$\theta_2 = T_2 - T_\infty = \theta_b \frac{\cosh(mz - mL)}{\cosh(mL)}. \quad (6)$$

, where

$$m = \sqrt{\frac{hp}{kA_c}} \quad (7)$$

and

$$\theta_b = \theta_{jos} - x\theta_{sus} = \theta_{jos} - \theta_{ext}. \quad (8)$$

Here p and  $A_c$  are the perimeter and the area of the electrode cross section. The heat load, in this case, is  $Q_2$ :

$$Q_2 = \theta_b \sqrt{kAhp} \tanh(mL) \quad (9)$$

The solution of this problem requires the following steps:

**Step 1.** The temperature at the upper boundary due to the diffusion of heat from the electrical discharge

$$\theta_2 \Big|_{z=L} = (\theta_{jos} - x\theta_{sus}) \frac{1}{\cosh(mL)} = \theta_{sus}(1-x) \quad (10)$$

We conclude that

$$\cosh(mL) = \frac{\theta_{jos} - x\theta_{sus}}{\theta_{sus}(1-x)} \quad (11)$$

or

$$m = \frac{1}{L} \operatorname{arccosh} \left[ \frac{\theta_{jos} - x\theta_{sus}}{\theta_{sus}(1-x)} \right] \quad (12)$$

From the definition of "m", we can determine the convective heat transfer coefficient, h:

$$h = \frac{m^2 k A_c}{p} \quad (13)$$

For a cylindrical electrode  $p = 2\pi r_0$  and  $A_c = \pi r_0^2$ . Consequently,

$$h = \frac{m^2 k r_0}{2} \quad (14)$$

**Step 2.** The value of the convective heat transfer coefficient obtained above is used in Eq. (4):

$$x \cdot \theta_{sus} = \frac{Q_1}{V} \frac{r_0}{2h} = \frac{Q_1}{\pi r_0^2 L} \frac{r_0}{2h} = \frac{Q_1}{2\pi r_0 L h} \quad (15)$$

Consequently,

$$Q_1 = 2\pi r_0 L h x \theta_{sus}. \quad (16)$$

**Step 3.** The heat that loads the electrode due to the electrical discharge is calculated using Eq. (9) and the numerical results obtained above.

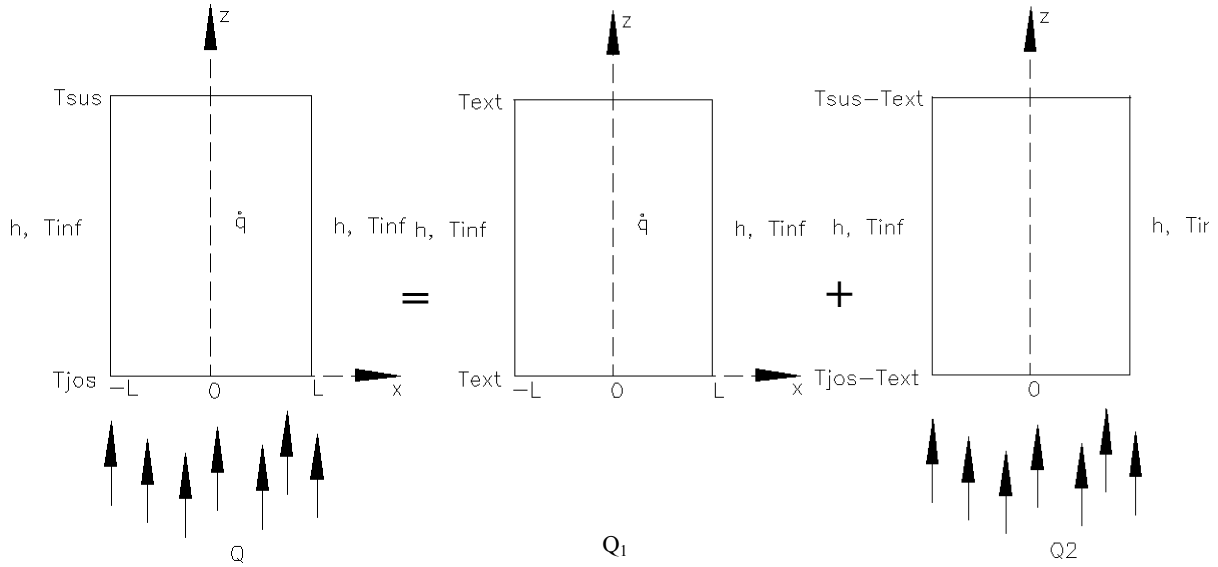
$$Q_2 = (\theta_{jos} - x\theta_{sus}) \pi r_0 \sqrt{2khr_0} \tanh(mL) \quad (17)$$

**Step 4.** The percentage of the power dissipated in the electrode is calculated considering the following:

- the used power:  $P_c = UI$  ;
- the dissipated power:  $P_d = Q_1 + Q_2$  ;
- the percentage of the power dissipated in the electrode:

$$P_{\%el} = \frac{P_d}{P_c} = \frac{Q_1 + Q_2}{UI}. \quad (18)$$

**2.2. Rectangular prism electrode**



**Fig. 2.** The temperature field in a prismatic electrode during the electroerosion process

The mathematical model of a prismatic electrode is similar to the case of the cylindrical electrode.

The temperature increase is due to the two thermal loads: the Joule effect uniformly distributed on the volume of the electrode,  $\dot{q}$  [W/m<sup>3</sup>] and the heat due to the electrical discharge.  $\theta = \theta_1 + \theta_2$ :

1. For an electrode with a rectangular cross-section (see Figure 2), the temperature  $T_1$  is:

$$T_1 = -\frac{\dot{q}x^2}{4k} + \frac{\dot{q}L^2}{4k} + \frac{\dot{q}L}{2h} + T_\infty; \tag{19}$$

$$\theta_1 = T_1 - T_\infty = -\frac{\dot{q}x^2}{4k} + \frac{\dot{q}L^2}{4k} + \frac{\dot{q}L}{2h}. \tag{20}$$

The increase of the temperature due to the Joule effect is considered as being a fraction "x" of the upper temperature increase:

$$\theta_{1,ext} = \frac{\dot{q}L}{2h} = x \cdot \theta_{sus} \tag{21}$$

, where  $\dot{q} = Q_1 / V$ . Here V is the volume of the electrode:  $V=2LIH$ , where 2L and H are the width and the height of the electrode, while l is the electrode dimension normal to Figure 2.

2. The second temperature,  $T_2$ , is due to the diffusion of the heat along the electrode and the convection of the heat from the electrode. The electrode is regarded as a fin with finite length and isolated tip [1]:

$$\theta_2 = T_2 - T_\infty = \theta_b \frac{\cosh(mz - mH)}{\cosh(mH)}, \tag{22}$$

where  $\theta_b$  can be calculated using Eq. (11). The heat flux at the base of the electrode [1]:

$$Q_2 = \theta_b \sqrt{kAhptgh(mH)}. \tag{23}$$

**Step 1.** The increase of the temperature due to the electrical discharge,  $\theta_{sus} \cdot (1-x)$ , is the temperature given by Eq. (21) for  $z = H$ :

$$\theta_{sus} \cdot (1-x) = (\theta_{jos} - x\theta_{sus}) \frac{1}{\cosh(mH)}. \tag{24}$$

We conclude that

$$m = \frac{1}{H} \arccos h \left[ \frac{\theta_{jos} - x\theta_{sus}}{\theta_{sus}(1-x)} \right]. \tag{25}$$

We can determine the convective heat transfer coefficient, h, using Eq. (13). For an electrode with a cross-section  $2L \times l$ ,  $p = 2(2L+l)$  and  $A_c = 2Ll$ . Consequently,

$$h = \frac{m^2 kLl}{2L+l}. \tag{26}$$

**Step 2.** The value h is further used by Eq. (21):

$$x \cdot \theta_{sus} = \frac{Q_1 L}{V h} = \frac{Q_1 L}{2LIH h} = \frac{Q_1}{2IHh}. \tag{27}$$

Consequently,

$$Q_1 = 2IHhx\theta_{sus}. \tag{28}$$

**Step 3.** The heat due to the electrical discharge:

$$Q_2 = (\theta_{jos} - x\theta_{sus}) \sqrt{hp k A_c t g h(mH)}. \tag{29}$$

**Step 4.** The percentage of the useful power that is lost due to the electrode heating can be found as follows:

- the consumed power:  $P_c = UI$  ;
- the power lost for the electrode heating:  $P_d = Q_1 + Q_2$  ;
- the percentage of the power lost due to the electrode heating:  $P_{\%,el} = \frac{P_d}{P_c} = \frac{Q_1 + Q_2}{UI}$  .

### 3. RESULTS AND DISCUSSIONS

The analysis has the variable "x" as the unknown. Following the mathematical model of Section 2, the steps 1-4, we can find the exact solution by verifying the first law of Thermodynamics:

$$\frac{dE}{dt} = Q_1 + Q_2 - Q_h, \tag{30}$$

where  $Q_h$  is the heat transfer lost by convection and  $dE/dt$  is the variation of the internal energy of the electrode.

#### 3.1. Cylindrical electrode

The new terms of Eq. (30) can be evaluated as follows:

$$Q_h = h(2\pi r_0 L)(T - T_{inf}) = h(2\pi r_0 L)\theta \tag{31}$$

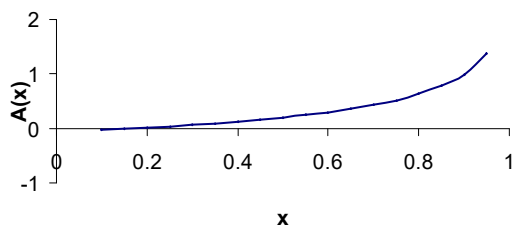
$$\frac{dE}{dt} = mc \frac{dT}{dt} = \rho(\pi r_0^2 L)c \frac{\theta}{t} \tag{32}$$

, where t is the time when the equilibrium temperature is attained and  $\theta$  was considered the upper boundary temperature increase,  $\theta_{sus}$ .

Combining the equations (30)-(32), we obtain the function A(x), Eq. (33), whose plot shows us the solution.

$$A(x) = Q_1 + Q_2 - \theta_{sus} \cdot (\pi r_0 L)(\rho r_0 c / t + 2h) \tag{33}$$

For the experimental data presented by Table 1, Fig. 3 presents the A(x) plot and reveals the solution as being  $x = 0.15$ . Consequently,  $Q_1=0.12W$ ,  $Q_2=0.18W$  and  $P_{\%,el}=0.16\%$ .



**Fig. 3.** The A(x) plot for a cylindrical electrode

Table 1. The cylindrical case experiment.

Nr. Crt.	Variable	Units of measurement	Value
1	L	[mm]	37.36
2	R <sub>0</sub>	[mm]	4.975
3	k	[W/mK]	111
4	ρ	[Kg/m <sup>3</sup> ]	8800
5	c	[J/Kg/K]	385
6	U	[V]	40
7	I	[A]	20
8	T <sub>sus</sub>	[°C]	18
9	T <sub>jos</sub>	[°C]	20
10	T <sub>∞</sub>	[°C]	17
11	t	[s]	20

#### 3.2. Rectangular prism electrode

The new terms of Eq. (30), in this case, are:

$$Q_h = h2(2L+l)H(T - T_{inf}) = h2(2L+l)H\theta \tag{34}$$

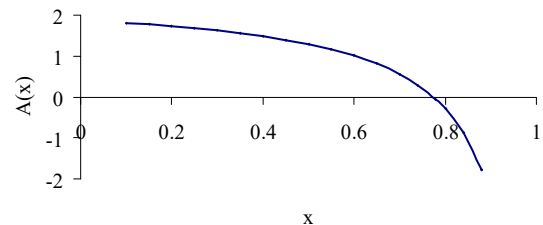
$$\frac{dE}{dt} = mc \frac{dT}{dt} = \rho(2LIH)c \frac{\theta}{t} \tag{35}$$

while the A(x) function was considered as follows:

$$A(x) = Q_1 + Q_2 - \theta_{sus}(2H)[\rho lLc / t + (2L+l)h] \tag{36}$$

Table 2. The rectangular prism case experiment.

Nr. Crt.	Variable	Unit of measurement	Value
1	L	[mm]	3.75
2	l	[mm]	13.8
3	H	[mm]	10.9
4	k	[W/mK]	398
5	ρ	[Kg/m <sup>3</sup> ]	8800
6	c	[J/Kg/K]	385
7	U	[V]	40
8	I	[A]	20
9	T <sub>sus</sub>	[°C]	18
10	T <sub>jos</sub>	[°C]	19
11	T <sub>∞</sub>	[°C]	17
12	t	[s]	20



**Fig. 4.** The A(x) plot for a prismatic electrode

In this experiment (see Table 2 and fig. 4), the solution is  $x = 0.78$ ,  $Q_1=10.98W$ ,  $Q_2=10.87W$  and  $P_{\%,el}=2.73\%$ .

### REFERENCES

[1] Neagu, M., *Thermal Phenomena at Materials Processing*, Tehnica-Info, Chişinău, Moldavia, 2002.