# Method to Calculate Oscillatory Die Volumetric Deformation Process Specific Parameters 

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#### Abstract

Deformation process by using an oscillatory die is more convenient than conventional volumetric forming, because it requires, to obtain the same piece, a smaller manufacturing force (in the conditions of a floating contact surface) and allows higher deformation degrees, due to a slower cold hardening process. The paper suggests a calculus method to evaluate the forming force and the torque applied to oscillatory shaft.


Keywords: orbital forming, floating contact surface.

## 1. Working Principle

During orbital volumetric forming (Figure 1), oscillatory die, 1, incrementally deforms worked piece 2 . The deformation zone, 4 , is mobile in space and time and runs over the whole worked piece frontal surface during an equipment functioning cycle. Superior die axis is inclined by $\theta$ angle, referred to the inferior die one.

During manufacturing process, O point slides along inferior die axis. Superior die rolling onto piece's surface is given by two rotation motions done around $Z_{1}$ and $Z_{2}$ axis. At a complete working cycle, piece's height reduction is equal to " $s$ " feed done by superior die along $Z_{2}$ axis.

From mechanical point of view, this ensemble part is acting as a rigid with a fix point.


Fig. 1 - Oscillatory Die Forming Process Kinematics:
1 - oscillatory die; 2 - manufactured piece; 3 - inferior die; 4 - floating contact surface.

## 2. Floating Contact Surface Curve Radius

Under some restrictions, we may consider that orbital volumetric forming process can be compared to the rolling process. This analogy is easier to be accepted when forming annular pieces with a very big radius interior hole. The equation of oscillatory dies conical active surface (Figure 1) is

$$
\begin{equation*}
X^{2}+Y^{2}-\frac{Z^{2}}{\operatorname{tg}^{2} \theta}=0 \tag{1}
\end{equation*}
$$

During rolling process, roles radius is constant. When forming by an oscillatory die, curve radius has a variation from $O$ center to the periphery and, as consequence, has an influence onto the forming force magnitude.

Through a certain point A, (Figure 2), placed on the generating line of the cone from XOZ plain, another plain, $\Delta$, having its normal parallel to this generating line, is considered.

The equation defining $\Delta$ plain is

$$
\begin{equation*}
X \cos \theta+Z \sin \theta=R_{A} \tag{2}
\end{equation*}
$$

where $\mathrm{R}_{\mathrm{A}}=$ OA denomination was made.


Fig. 2 - The Co-ordinates System Attached to the Oscillatory Die

By eliminating " $X$ " between equations (1) and (2) and by considering $\operatorname{tg}^{4} \theta \approx 0$ and $\cos \theta \approx 1$ ( $\theta$ being very small), it follows

$$
\frac{\left(Z+R_{A} \operatorname{tg}^{3} \theta\right)^{2}}{R_{A}^{2} \operatorname{tg}^{2} \theta}-\frac{Y^{2}}{R_{A}^{2}}=1
$$

respective

$$
\begin{equation*}
Z=\left(Y^{2}+R_{A}^{2}\right)^{\frac{1}{2}} \operatorname{tg} \theta-R_{A} \operatorname{tg}^{3} Y \tag{3}
\end{equation*}
$$

Second and third order derivatives are:

$$
\begin{align*}
& Z^{\prime}=Y\left(Y^{2}+R_{A}^{2}\right)^{-\frac{1}{2}} \operatorname{tg} \theta \text { and }  \tag{4}\\
& Z^{\prime \prime}=\left[\left(Y^{2}+R_{A}^{2}\right)^{-\frac{1}{2}}-Y^{2}\left(Y^{2}+R_{A}^{2}\right)^{-\frac{3}{2}}\right] \operatorname{tg} \theta
\end{align*}
$$

Thus, contact surface between oscillatory die and piece curve radius equation will be

$$
\begin{equation*}
\rho^{*}=\left|\frac{\left(1+z^{\prime}\right)^{1,5}}{z^{\prime \prime}}\right| \tag{6}
\end{equation*}
$$

Worked piece's superior surface may be considered a plain having the equation

$$
\begin{equation*}
\frac{Z \cos \theta}{s}-\frac{X \operatorname{tg} \theta}{s \cos \theta}=1 \tag{7}
\end{equation*}
$$

where "s" means oscillatory die feed, [mm / osc.]. The solution of the equations system (2) and (7) is

$$
\begin{gather*}
Y^{2}=\frac{s^{2}}{\operatorname{tg} \theta \cos ^{2} \theta}+\frac{2 s X}{\operatorname{tg} \theta \cos ^{3} \theta}-  \tag{8}\\
-\left(\frac{1}{\cos ^{4} \theta}-1\right) X^{2}
\end{gather*}
$$

If $\cos \theta \approx 1$, it follows

$$
\begin{equation*}
Y= \pm \frac{s}{\operatorname{tg} \theta} \sqrt{\frac{2 X \operatorname{tg} \theta}{s}+1} \tag{9}
\end{equation*}
$$

From equation (6), when $\mathrm{Y}=0$, it results

$$
\begin{equation*}
\rho_{A}^{*}=\frac{R_{A}}{\operatorname{tg} \theta} \tag{10}
\end{equation*}
$$

Because die conical surface is convex, only positives solutions of equation (9) are considered. After making the substitution $\mathrm{X}=\mathrm{R}_{\mathrm{A}}$, the expression

$$
\begin{align*}
Y & =\frac{s}{\operatorname{tg} \theta} \sqrt{\frac{2 R_{A} \operatorname{tg} \theta}{s}+1}= \\
& =2 R_{A} Q_{A} \sqrt{\frac{1}{Q_{A}}+1}, \tag{11}
\end{align*}
$$

results, where

$$
\begin{equation*}
Q_{A}=\frac{s}{2 R_{A} \operatorname{tg} \theta} . \tag{12}
\end{equation*}
$$

By substituting Y, from (11), into relation (6), curve radius corresponding to " B " point results:

$$
\begin{align*}
\rho_{B}^{*} & =\frac{R_{A}\left(2 Q_{A}+1\right)^{3}}{\operatorname{tg} \theta} . \\
& \cdot\left[1+\frac{4 Q_{A}\left(Q_{A}+1\right) \operatorname{tg}^{2} \theta}{\left(2 Q_{A}+1\right)^{2}}\right]^{\frac{3}{2}} .
\end{align*}
$$

It may be considered that arithmetic average of curve radii in points " $A$ " and " $B$ " gives AB arc curve radius

$$
\begin{equation*}
\rho_{A B}^{*}=\frac{1}{2}\left(\frac{R_{A}}{\operatorname{tg} \theta}+\rho_{B}^{*}\right) . \tag{14}
\end{equation*}
$$

## 3. Forming Force and Torque Applied to Oscillatory Shaft

To initiate the forming process, it is necessary that the active force reaches its critical value, calculated with relation

$$
\begin{equation*}
F_{o}=k A_{c} \sigma_{o}, \tag{15}
\end{equation*}
$$

where:
Ac means oscillatory die - worked piece contact area surface;
k - factor whose value depends on forming speed magnitude, die geometrical complexity, friction ratio between oscillatory die and piece etc.;
$\sigma_{0}$ - material flowing limit.
Contact surface can be found, under some approximations, in conformity to those presented in [4], chapter 3. In this sense, CDBE curve equation (Figure 3), conform to (9) relation, is

$$
\begin{equation*}
Y=\frac{s}{\operatorname{tg} \theta} \sqrt{\frac{2 X \operatorname{tg} \theta}{s}+1} . \tag{16}
\end{equation*}
$$

The equation of circle that delimitates worked piece's frontal surface is

$$
\begin{equation*}
X^{2}+Y^{2}=R^{2} . \tag{17}
\end{equation*}
$$

Solving the system given by (16) and (17), leads to the following solutions:


Fig. 3 - Mark Projection on Worked Piece Frontal

The surface of mark projection can be calculated by using the relation

$$
\begin{align*}
A_{C} & =\int_{X_{c}}^{x_{E}} \frac{s}{\operatorname{tg} \theta} \sqrt{\frac{s X \operatorname{tg} \theta}{s}+1} d x+  \tag{20}\\
& +\frac{1}{2}\left(\frac{s}{\operatorname{tg} \theta}\right)^{2} \sqrt{\frac{2 \operatorname{Rtg} \theta}{s}-1} .
\end{align*}
$$

If a polar co-ordinates system by " O " origin is attached to the worked piece, by mathematical transformations, CDE curve equation will become

$$
\begin{equation*}
\rho=\frac{s}{2 \operatorname{tg} \theta(1-\cos \beta)}=\frac{2 R Q}{1-\cos \beta}, \tag{21}
\end{equation*}
$$

where $\rho$ is the polar radius;

$$
Q=\frac{s}{2 \operatorname{Rtg} \theta}, \text { vezi formula (12); }
$$

$\beta$ - the angle at the center corresponding to the mobile point on the circle having " $O$ " as center and " $R$ " radius.

When

$$
\alpha=\arccos \left(1-\frac{s}{R \operatorname{tg} \alpha}\right)
$$

the
projection of contact surface between oscillatory die and worked piece can be calculated by using the formula

$$
\begin{align*}
A_{C} & =\int_{0}^{R} \int_{0}^{\alpha} \rho d \beta d \rho+\int_{0}^{\pi} \frac{s}{\frac{\operatorname{tg} \theta(1-\cos \beta)}{}} \int_{0} \rho d \beta d \rho=  \tag{22}\\
& =\frac{R^{2}}{2} \alpha+\frac{1}{6} R^{2}\left(1+\frac{s}{R_{A} \operatorname{tg} \theta}\right) \sin \alpha .
\end{align*}
$$

If relation (12) is used, equation (22) becomes

$$
\begin{equation*}
A_{C}=\frac{R^{2}}{2}\left[\alpha+\frac{(1+2 Q) \sin \alpha}{3}\right] \tag{23}
\end{equation*}
$$

## 3. Calculation of Torque Applied to Oscillatory Shaft

Conform to Figure 4, the expression to calculate the torque applied to oscillatory shaft is

$$
\begin{equation*}
M=F o \cdot d, \tag{24}
\end{equation*}
$$

where: $\mathrm{F}_{\mathrm{o}}$ is the orbital forming force, [daN]; d - the brace of the force (the distance between resultant direction and shaft rotation axis).
Dedicated literature, [2], shows that force's direction during orbital forming is placed to the distance $\mathrm{R} / 2$ relative to worked piece axis and at the middle of the contact arc, respective $\gamma / 2$ (Figure 4).

From formula (21), it follows

$$
\begin{equation*}
\cos \beta=1-\frac{2 R Q}{\rho} . \tag{25}
\end{equation*}
$$

The value of $\gamma$ angle results from relation (25) if $\rho=\mathrm{R} / 2$

$$
\begin{equation*}
\gamma=\arccos (1-4 Q) . \tag{26}
\end{equation*}
$$

Conform to (14), contact between oscillatory die surface and worked piece arc
radius value is very big, compared to die feed. As consequence, it results

$$
d=\frac{R}{2} \sin \frac{\gamma}{2}=\frac{R}{2} \sin \frac{\arccos (1-4 Q)}{2},[\mathrm{~m}] .(27)
$$

Thus, the expression of torque applied to oscillatory shaft has the form

$$
\begin{equation*}
M=\frac{F_{0} R}{2} \sin \frac{\arccos (1-4 Q)}{2},[\mathrm{daNm}] . \tag{28}
\end{equation*}
$$



Fig. 4 - Orbital Forming Force Direction Co-ordinates

## References

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## Rezumat

Procesul de deformare cu matriță oscilantă este mai avantajos decât presarea volumică convențională deoarece necesită, pentru obținerea aceleiaşi piese, o forță de prelucrare mai mică (în condițiile unei suprafețe de contact flotante) şi permite grade de deformare mai mari datorită unui proces de ecruisare mai lent. Lucrarea propune o metodă de calcul pentru forța de deformare şi momentul de rotație aplicat arborelui oscilant.

## Résumé

Le processus de déformation avec une matrice oscillante est plus avantageux que la déformation volumique conventionnelle parce que nécessite, pour obtenir la même pièce, une force d'usinage plus réduite (dans les conditions d'une surface de contact flottante) et permet degrés de déformation plus élevées, a la cause d'un écrouissage plus lente. Ce papier propose une méthode pour calculer la force de déformation et le moment de rotation appliqué au arbre oscillant.

