# The Profiling of the Abrasive Tools for the Continuous Sharpening of Hobbing Cutters with Shifted Teeth 

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#### Abstract

In this paper, are presented simplified methods for the profiling of the second-order tools used for the continuous generation of the back surfaces of the worm hob with shifted teeth.

The solving of the technical problems related to the profiling of the abrasive tools used for the continuous generation of the back surfaces of the worm tool's teeth has led to the replacement of the theoretical secondorder tool with a ruled revolution surface.

The technological solutions for the approximation of the suggested abrasive tools' profile, of disc or end mill type, show the fact that the size of the approximation error is very small and can be technically accepted.


Keywords: second-order tool's profiling, hobbing cutter, continuous sharpening.

## 1. Simplified profiling methods of the tools used for the continuous generation of the helical back surfaces

The continuous generation of the helical back surfaces of the hobbing cutter's teeth can be made by approximate methods using tools bounded by revolution surfaces (of disc or end mill type), accepting a sufficiently small error.

The condition for determining the characteristic curve in the helical surface's generation can be approximated, through the condition that the intersection line of the plane normal to the helix with the contact circle's plane intersects the second degree tool's axis. "the contact condition".

### 1.1. The second-order tool's profiling methodic

In the following text, the use of the approximate method for profiling the disc or end mill tools is shown for the case of the worm tool's helical Archimede's back surface's generation.

### 1.2. The profiling of the disc tool

In the XYZ reference system (fig.1), the equations of the helical line from the cylinder of variable R radius are:

$$
\left\{\begin{array}{l}
X=R \cdot \cos \varphi  \tag{1}\\
Y=R \cdot \sin \varphi \\
Z=u+p \cdot \varphi
\end{array}\right.
$$

where: $\varphi$ is the variable angular parameter;
$u$ - variable parameter, measured in the helical surface's axial direction;
p - the surface's helical parameter.
Equations (1) represent a family of helical lines - a general way to express the form of a cylindrical helical surface of constant pitch.

Through the transformation
$\left\|\begin{array}{l}X_{1} \\ Y_{1} \\ Z_{1}\end{array}\right\|=\left\|\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \omega_{e} & \sin \omega_{e} \\ 0 & -\sin \omega_{e} & \cos \omega_{e}\end{array}\right\|\left[\left\|\begin{array}{l}R \cdot \cos \varphi \\ R \cdot \sin \varphi \\ u+p \cdot \varphi\end{array}\right\|-\left\|\begin{array}{c}0 \\ 0 \\ u_{0}\end{array}\right\|\right]$
the helical line's equations are related to the $\mathrm{X}_{1} \mathrm{Y}_{1} \mathrm{Z}_{1}$ system, joined to the disc tool.

After development, the family of helical lines results,

$$
\left\{\begin{array}{l}
X_{1}=R \cdot \cos \varphi  \tag{3}\\
Y_{1}=R \cdot \sin \varphi \cdot \cos \omega_{e}+\left(u-u_{0}+p \cdot \varphi\right) \cdot \sin \omega_{e} \\
Z_{1}=-R \cdot \sin \varphi \cdot \sin \omega_{e}+\left(u-u_{0}+p \cdot \varphi\right) \cdot \cos \omega_{e}
\end{array}\right.
$$

The plane normal to the helix has the equation:
$-\left[X_{1}-R \cdot \cos \varphi\right] R \cdot \sin \varphi+\left[Y_{1}-R \cdot \sin \varphi \cdot \cos \omega_{e}-\left(u-u_{0}+p \cdot \varphi\right) \sin \omega_{e}\right]$.
$\cdot\left(R \cdot \cos \omega_{e} \cdot \cos \varphi+p \cdot \sin \omega_{e}\right)+\left[Z_{1}+R \cdot \cos \varphi \cdot \sin \omega_{e}-\left(u-u_{0}+p \cdot \varphi\right) \cos \omega_{e}\right]$.
$\cdot\left(-R \cdot \sin \omega_{e} \cdot \cos \varphi+p \cdot \cos \omega_{e}\right)=0$

The contact circle's plane, of the disc tool's peripheral surface with the helix correspondent to the M point, is:

$$
\begin{align*}
& Z_{1}=H=-R \cdot \sin \varphi \cdot \sin \omega_{e}+ \\
& +\left(u-u_{0}+p \cdot \varphi\right) \cdot \cos \omega_{e} \tag{5}
\end{align*}
$$

Also, the axis of the tool's rotation surface being defined:

$$
A\left\{\begin{array}{l}
X_{1}=a  \tag{6}\\
Y_{1}=0
\end{array}\right.
$$

the assembly of equations (4), (5) and (6) represent "the contact condition":

$$
\begin{align*}
& -[a-R \cdot \cos \varphi] R \cdot \sin \varphi+ \\
& +\left[-R \cdot \sin \varphi \cdot \cos \omega_{e}-\left(u-u_{0}+p \cdot \varphi\right) \sin \omega_{e}\right] .  \tag{7}\\
& \cdot\left(R \cdot \cos \omega_{e} \cdot \cos \varphi+p \cdot \sin \omega_{e}\right)=0
\end{align*}
$$

Condition (7) has a certain universality character, keeping its form with the change of the helical surface's type.

The coordinates of the disc tool's axial section will be:

$$
\left\{\begin{array}{l}
R_{0}=\left\{[R \cos \varphi-a]^{2}+\left[R \sin \varphi \cos \omega_{e}+\right.\right.  \tag{8}\\
\left.\left.+\left(u-u_{0}+p \varphi\right) \sin \omega_{e}\right]^{2}\right\}^{\frac{1}{2}} \\
H=-R \sin \varphi \sin \omega_{e}+ \\
+\left(u-u_{0}+p \varphi\right) \cos \omega_{e} .
\end{array}\right.
$$

section will
together with the contact condition (7).
In the following text, the use of the approximate method for profiling the disc tool is shown for the case of the helical Archimedes's surface's generation (fig.1).


Fig. 1 The generation of the Archimedes's worm
The helical line's equations are:

$$
\left\{\begin{array}{l}
X=R \cdot \cos \varphi  \tag{9}\\
Y=R \cdot \sin \varphi \\
Z=u+p \cdot \varphi
\end{array}\right.
$$

with:

$$
\begin{equation*}
u=u_{1} \cdot \sin \alpha_{0}-b=R \cdot \operatorname{tg} \alpha_{0}-b \tag{10}
\end{equation*}
$$

and the variation limits:

$$
\begin{align*}
& u_{\min }=\frac{R_{\mathrm{i}}}{\cos \alpha_{0}}  \tag{11}\\
& u_{\max }=\frac{R_{e}}{\cos \alpha_{0}}
\end{align*}
$$

The contact condition is calculated, according to (7) where:

$$
\begin{equation*}
\omega_{e}=\operatorname{arctg} \frac{P_{E}}{2 \cdot \pi \cdot R_{e}} \tag{12}
\end{equation*}
$$

The coordinates of the disc tool's axial section become:

$$
G:\left\{\begin{array}{l}
R_{0}=\left\{[R \cos \varphi-a]^{2}+\left[R \sin \varphi \cos \omega_{e}+\right.\right.  \tag{13}\\
\left.\left.+\left(u-u_{0}+p \varphi\right) \sin \omega_{e}\right]^{2}\right\}^{\frac{1}{2}} \\
H=-R \sin \varphi \sin \omega_{e}+ \\
+\left(u-u_{0}+p \varphi\right) \cos \omega_{e} .
\end{array}\right.
$$

### 1.1.2. The profiling of the end mill tool

The methodic shown in the above text gains resolution in the profiling of the end mill tool, too, where the error of the profile determined with the approximate method is very small.

It is accepted, as shown in [1], [2], the replacement of the enwrapping condition with a tool-half-finished material touching condition.

Taking into account equations (1), on the current point on the helix, the equation of the plane normal to the helix is defined:

$$
\begin{align*}
& -(X-R \cdot \cos \varphi) \cdot R \cdot \sin \varphi+ \\
& +(Y-R \cdot \sin \varphi) \cdot R \cdot \cos \varphi+  \tag{14}\\
& +(Z-u+p \cdot \varphi) \cdot p=0
\end{align*}
$$

The level plane's equation corresponding to the M point is:

$$
\begin{align*}
& \Delta_{H}:-H \cdot \sin \varphi+Y \cdot \cos \varphi- \\
& -(Z-u+p \cdot \varphi) \frac{p}{R}=0 \tag{15}
\end{align*}
$$

The ensemble of (1), (14) and (15) equations determine the intersection line of the plane normal to the helix with the $\mathrm{X}=\mathrm{H}$ plane,

$$
\begin{equation*}
X=H \tag{16}
\end{equation*}
$$

The intersection condition of the $\Delta_{\mathrm{H}}$ straight line with the end mill surface's axis, bearing the equations:

$$
\begin{align*}
& \Delta_{H}:-H \cdot \sin \varphi+Y \cdot \cos \varphi- \\
& -(Z-u+p \cdot \varphi) \frac{p}{R}=0 \tag{17}
\end{align*}
$$

determines "the contact condition"

$$
A\left\{\begin{array}{l}
X=0  \tag{18}\\
Z=a
\end{array}\right.
$$

The coordinates of the end mill tool's axial section are:

$$
R \cdot \cos \varphi \cdot \sin \varphi+(a-u+p \cdot \varphi) \frac{p}{R}=0
$$

The results of the calculus for the end mill tool's profiling, shown in figure 3, accentuates the fact that the error of the profile determined with the suggested method is very small and can be accepted from the technical point of view.

## 2. Technological solution for the replacement of the profile of the tools used for the continuous generation of the hobbing cutter's helical back surfaces

To solve the technical problems related to the continuous generation of the hobbing cutter's teeth back surfaces the replacement of the theoretical second degree tool with a ruled revolution surface of an abrasive tool is suggested.

The tool's revolution surface obtained with the classical method from the theory of enwrapping surfaces can be approximated by a hyperboloid (fig.2), generated by the revolution of the $\Delta$ straight line, which makes the $\lambda$ angle with the rotation axis. Considering $r_{0}$ as the measured distance on the common perpendicular, between the $\Delta$ line and the hyperboloid's axis.


Fig. 2 Replacing hyperboloids
In the XYZ reference system, the hyperboloid's equations are:

$$
\left\{\begin{array}{l}
X=r_{0} \cdot \cos \varphi-u \cdot \sin \lambda \cdot \sin \varphi  \tag{20}\\
Y=r_{0} \cdot \sin \varphi+u \cdot \sin \lambda \cdot \cos \varphi \\
Z=u \cdot \cos \lambda+a
\end{array}\right.
$$

with $u$ and $\varphi$ variable parameters; and $a$ is the constant which determines the replacing surface's position towards the hobbing cutter's axis.

The intersection hyperbola with the ZY plane, the axial plane of the revolution surface, has the equation:

$$
\begin{equation*}
(Z-a)^{2} \cdot \operatorname{tg}^{2} \lambda=Y^{2}-r_{0}^{2} \tag{21}
\end{equation*}
$$

The resolution of the suggested problem, the replacement of the theoretical tool's axial section with the hyperbola (21), expects the determination of the parameters: $\mathrm{a}, \lambda$ and $\mathrm{r}_{0}$

The condition that the two curves coincide in three points is necessary, figure 3. Using the notations: $\left(\mathrm{Z}_{\mathrm{E}}, \mathrm{Y}_{\mathrm{E}}\right),\left(\mathrm{Z}_{\mathrm{P}}, \mathrm{Y}_{\mathrm{P}}\right),\left(\mathrm{Z}_{\mathrm{I}}, \mathrm{Y}_{\mathrm{I}}\right)$ for the coordinates of the three points, from equation (21) the hyperboloid's parameters result:


Fig. 3 The replacing tool's axial section

$$
\begin{align*}
& a=\frac{Z_{E}^{2}-Z_{I}^{2}-k \cdot\left(Z_{P}^{2}-Z_{I}^{2}\right)}{2 \cdot\left[Z_{E}-Z_{I}-k \cdot\left(Z_{P}-Z_{I}\right)\right]}  \tag{22}\\
& \operatorname{tg} \lambda=\sqrt{\frac{Y_{E}^{2}-Y_{I}^{2}}{\left(Z_{E}-a\right)^{2}-\left(Z_{I}-a\right)^{2}}}  \tag{23}\\
& r_{0}=\sqrt{Y_{I}^{2}-\left(Z_{I}-a\right)^{2} \cdot \operatorname{tg}^{2} \lambda} \tag{24}
\end{align*}
$$

where:

$$
\begin{equation*}
k=\frac{Y_{E}^{2}-Y_{I}^{2}}{Y_{P}^{2}-Y_{I}^{2}} \tag{25}
\end{equation*}
$$

The $Y_{E}, Z_{E}, \ldots, Y_{I}, Z_{I}$ coordinates are determined in points belonging to the theoretical second-order tool's axial profile.

## 3. The theoretical generation error

Starting from the approximation of the theoretical axial profile with replacing hyperbolae (21) the generation error is calculated as a difference between the coordinates of the pairs of points placed on the two curves at equal distance towards the hob's axis.

$$
\begin{equation*}
\varepsilon=\mathrm{Z}_{\mathrm{T}}-\mathrm{Z}_{\Delta} \tag{26}
\end{equation*}
$$

In figures 4 and 5 it is shown an example of applying this method to the profiling of a worm hob with continuous sharpening having the dimensions: $\mathrm{m}=5 \mathrm{~mm}, \mathrm{R}_{\mathrm{es}}=60 \mathrm{~mm}, \alpha_{\mathrm{d}}=$ $20^{\circ}$.


Fig. 4 The flank's theoretical profile


Fig. 5 The flank's effectively generated profile Conclusions

1. The resolution of the technical problems related to the profiling of the abrasive tool for the continuous generation of the back surfaces has led to the replacement of the theoretical second degree tool with a ruled revolution surface.
2. The suggested technological solutions for the approximation of the disc or end mill type abrasive tool's profile have marked out the fact that the magnitude of the approximation error is very small and can be accepted from the technical point of view.

## Bibliography

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Profilarea sculelor abrazive pentru ascuțirea continuă a frezelor melc cu dinți decalați

- Rezumat -

În lucrare se prezintă metode simplificate de profilare a sculelor de ordinul doi pentru generarea continuă a suprafețelor de aşezare elicoidale ale frezei melc modul cu dinți decalați

Rezolvarea problemelor tehnice legate de profilarea sculelor abrazive pentru generarea continuă a suprafețelor de aşezare ale dinților sculei melc a condus la înlocuirea sculei teoretice de ordinul doi cu o suprafață de revoluție riglată.

Soluțiile tehnologice de aproximare a profilului sculelor abrazive de tip disc sau cilindro-frontale propuse evidențiază faptul că mărimea erorii de aproximare este foarte redusă şi poate fi acceptată din punct de vedere tehnic.

## Le profiler des outils abrasifs pour l'affilage continu <br> de fraise hélicoïdale avec les dents changées

## - Résumé -

Dans cet article on présente des méthodes simplifiées de profilation des outils de deuxième ordre pour la génération continues des surfaces en dépouille de la fraise-vis module avec dents décalées.

La solution des problèmes théoriques concernant la profilation des outils abrasifs pour la génération continue des surfaces en dépouille des dents de l'outil module a conduit au remplacement de l'outil théorique de deuxième ordre par une surface de révolution réglé.

Les solutions technologiques d'approximation du profil des outils abrasifs de type disque ou cylindriques ŕ deux tailles proposées, mettent en

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évidence le fait que la grandeur de l'erreur d'approximation est très réduite et peut être acceptée du point de vue technique.

