# The Modeling of the Generation of Conical Polyform Surfaces 

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#### Abstract

This paper suggests different methods for the modeling of a joint between two conical polyform surfaces, one enclosed and the other one enclosing. These methods can be easily applied on the universal grinding machines and have the advantage of being simple solutions that lead to a higher precision in the generation of the polyform surfaces. Keywords: modeling, polyform surfaces.


## 1. The Generation of Enclosed Conical Polyform Surfaces Using a Plane Generating Surface

## - Reference Systems and Generation Motions

In figure 1 , there are represented the reference systems towards which the generating plane and the polyform surface are referred.

The surface to be generated has a planetary motion which results from the combination of the motions around $\Delta_{1}$ and $\Delta_{2}$ axes.

The generating plane, P , is defined as an inclined plane with the $\alpha$ angle from the z axis and situated at the $\mathrm{A}_{12}$ distance from this axis, distance which is measured in a cross section on the generated shaft, the same with the $\xi \eta$ plane ( $\zeta=0$ ).


Fig. 1. Reference systems
The following reference systems are defined:

- XYZ is a mobile reference system with the Z axis over the axis of the planetary motion, $\Delta_{1}$;
- $x_{0}^{\prime} y_{0}^{\prime} z_{0}^{\prime}$ - a mobile reference system, identical to the XYZ system, which is in a rotation motion around the $\Delta_{1}$ axis;
- $\mathrm{x}_{0} \mathrm{y}_{0} \mathrm{z}_{0}$ - a fix reference system, situated in the generating plane - the front surface of the abrasive tool;
- zyz - a fix reference system, with the $z$ axis over the planetary motion's axis, $\Delta_{1}$;
- $X_{1} Y_{1} Z_{1}$ - a fix reference system, coinciding with the $\mathrm{x}_{0} \mathrm{y}_{0} \mathrm{z}_{0}$ system.

The positions of the reference systems are also defined:

- e is the excentricity;
- $\mathrm{A}_{12}$ - the distance between the z and $\mathrm{z}_{0}$ axes, in the xy system;
- D/2 - the distance between the $\xi \eta \zeta$ and $\mathrm{x}_{0} \mathrm{y}_{0} \mathrm{z}_{0}$ systems' origins, in the xy system, related to the characteristic dimension of the polyform shaft in this plane;
- $\alpha$ - the taper angle of the straight generating line of the polyform surface.

The following correlation is obvious:

$$
\begin{equation*}
A_{12}=e+\frac{D}{2} \tag{1}
\end{equation*}
$$

- The Kinematics of the Generation Process

The conical polyform surface is generated in the planetary motion: the rotations by $\varphi_{1}$ and $\varphi_{2}$ angles around the $\Delta_{1}$ and $\Delta_{2}$ axes.

Thus, the rotation of the $\xi \eta \zeta$ system around $z_{0}^{\prime}$ axis is shown by the transformation:

$$
\begin{equation*}
x_{0}^{\prime}=\omega_{3}^{T}\left(-\varphi_{2}\right) \xi . \tag{2}
\end{equation*}
$$

Also, the rotation of the XYZ system around the Z axis by $\varphi_{1}$ angle has the following form:

$$
\begin{equation*}
x=\omega_{3}^{T}\left(-\varphi_{I}\right) X . \tag{3}
\end{equation*}
$$

The position of the mobile reference systems is defined by

$$
\begin{equation*}
X=x_{0}^{\prime}-e \tag{4}
\end{equation*}
$$

with

$$
e=\left\|\begin{array}{c}
e  \tag{5}\\
0 \\
0
\end{array}\right\| .
$$

The position of the fix reference systems is also defined

$$
\begin{equation*}
X_{I}=x-A \tag{6}
\end{equation*}
$$

with

$$
A=\left\|\begin{array}{c}
-A_{12}  \tag{7}\\
0 \\
0
\end{array}\right\| .
$$

Thus it results the absolute motion

$$
\begin{equation*}
X_{I}=\omega_{3}^{\mathrm{T}}\left(\varphi_{I}\right)\left[\omega_{3}^{\mathrm{T}}\left(-\varphi_{2}\right) \xi-e\right]-A, \tag{8}
\end{equation*}
$$

of the mobile reference system of the generated surface, $\xi \eta \zeta$, towards the $\mathrm{X}_{1} \mathrm{Y}_{1} \mathrm{Z}_{1}$ system and the inverse is defined as

$$
\begin{equation*}
\xi=\omega_{3}\left(-\varphi_{2}\right)\left[\omega_{3}\left(\varphi_{1}\right)\left[X_{1}+A\right]+e\right] . \tag{9}
\end{equation*}
$$

## - The family of generating planes

In the (9) motion, the generating plane, $P$, describes a family of planes whose envelope is the conical surface of the generated polyform shaft.

If in the $\mathrm{x}_{0} \mathrm{y}_{0} \mathrm{z}_{0}$ system, the generating plane is defined by:

$$
P\left\{\begin{array}{l}
x_{o}=0  \tag{10}\\
y_{o}=u \\
z_{o}=t
\end{array}\right.
$$

with $u$ and $t$ independent variables. Through the transformation of coordinates:

$$
X_{l}=\alpha x_{0}, \text { with } \alpha=\left\|\begin{array}{ccc}
\cos \alpha & 0 & -\sin \alpha  \tag{11}\\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{array}\right\|,
$$

the parametrical equations of the generating plane are determined in the $X_{1} Y_{1} Z_{1}$ system:

$$
P\left\{\begin{array}{l}
X_{l}=-t \sin \alpha  \tag{12}\\
Y_{l}=u \\
Z_{l}=t \cos \alpha
\end{array}\right.
$$

In (11) and (12), where $\alpha$ is the taper angle of the generating plane, through development, the surface family in the $\xi \eta \zeta$ system is determined:
$\|\xi\| \begin{aligned} & \eta \\ & \eta \\ & \zeta\end{aligned}\|=\| \begin{array}{ccc}\cos \varphi_{2} & -\sin \varphi_{2} & 0 \\ \sin \varphi_{2} & \cos \varphi_{2} & 0 \\ 0 & 0 & l\end{array} \|$.
$\cdot\left\{\left\|\begin{array}{ccc}\cos \varphi_{1} & \sin \varphi_{l} & 0 \\ -\sin \varphi_{l} & \cos \varphi_{1} & 0 \\ 0 & 0 & 1\end{array}\right\|\left[\left\|\left\|\begin{array}{c}X_{l}\end{array}\right\|-\right\| \begin{array}{c}-A_{12} \\ Y_{1} \\ Z_{l}\end{array}\|+\| \begin{array}{c}\| \\ 0 \\ 0\end{array} \|\right]+\| \begin{array}{c}e \\ \theta \\ \theta \|\end{array}\right\}$
or, after development:

$$
(P)_{\varphi_{1}}\left\{\begin{align*}
\xi= & {\left[X_{1}-A_{l 2}\right] \cos \left(\varphi_{1}-\varphi_{2}\right)+}  \tag{14}\\
& +Y_{1} \sin \left(\varphi_{1}-\varphi_{2}\right)+e \cos \varphi_{2} \\
\eta= & -\left[X_{1}-A_{12}\right] \sin \left(\varphi_{1}-\varphi_{2}\right)+ \\
& +Y_{1} \cos \left(\varphi_{1}-\varphi_{2}\right)+e \sin \varphi_{2} \\
\zeta= & Z_{1} .
\end{align*}\right.
$$

The (14) equations mean a family of surfaces, see (12), in the $\xi \eta \zeta$ system and, if they are related exclusively to (12), they bear the form:
$(P)_{\varphi_{1}}\left\{\begin{aligned} \xi= & {\left[-t \sin \alpha-A_{12}\right] \cos (1-i) \varphi_{1}+} \\ & +u \sin (1-i) \varphi_{1}+e \cos \left(i \varphi_{1}\right) ; \\ \eta= & {\left[-t \sin \alpha-A_{12}\right] \sin (1-i) \varphi_{1}+} \\ & +u \cos (1-i) \varphi_{l}+e \sin \left(i \varphi_{1}\right) ; \\ \zeta= & t \cos \alpha .\end{aligned}\right.$
The relationship between the rotation angles, $\varphi_{1}$ and $\varphi_{2}$, is defined:
$\frac{\varphi_{2}}{\varphi_{1}}=i$ - the transmission ratio.
Usually, the transmission ratio has the following values:

$$
\begin{equation*}
i=\frac{1}{2} ; \frac{1}{3} ; \frac{1}{4} . \tag{17}
\end{equation*}
$$

The envelope of the family of planes, (15), represents the generated polyform surface. Observation: this generation method can not be applied for holes, except for the following case: the P plane is a straight line.

For $\mathrm{t}=0$ and $\mathrm{u}=0$, in the cross section of the polyform profile the generating point is determined, the $(P)_{\varphi_{1}}$ family results:

$$
\left\lvert\, \begin{align*}
& \xi=-A_{12} \cos (1-i) \varphi_{1}+e \cos \left(i \varphi_{1}\right) ; \\
& \eta=A_{12} \sin (1-i) \varphi_{1}+e \sin \left(i \varphi_{1}\right) ;  \tag{18}\\
& \zeta=0 .
\end{align*}\right.
$$

The previous case belongs to the generation with a point, meaning the boring process.

## - The enwrapping condition

As it is known, the envelope of the $(\mathrm{P})_{\varphi 1}$ family is obtained by the combination of the family equations with the enwrapping equation, written for the profile in the plane:

$$
\begin{equation*}
\zeta=\mathrm{H}(\mathrm{H}-\operatorname{arbitrary} \text { value }) \tag{19}
\end{equation*}
$$

which, taking into account (15), determines the relationship:

$$
\begin{equation*}
t=\frac{H}{\cos \alpha} \tag{20}
\end{equation*}
$$

Thus, in the method of "plane generating trajectories", for the specific enwrapping condition,

$$
\begin{aligned}
\frac{\xi_{\varphi_{1}}^{\prime}}{\xi_{u}^{\prime}}= & \frac{\eta_{\varphi_{1}}^{\prime}}{\eta_{u}^{\prime}} ; \\
\xi_{\varphi_{i}}^{\prime}= & -\left[-t \sin \alpha-A_{12}\right](1-i) \sin \left[(1-i) \varphi_{1}\right]+ \\
& +u(1-i) \cos \left[(1-i) \varphi_{1}\right]-e \cdot i \sin \left(i \varphi_{1}\right) ; \\
\eta_{\varphi_{1}}^{\prime}= & -\left[-t \sin \alpha-A_{12}\right](1-i) \cos \left[(1-i) \varphi_{I}\right]+ \\
& +u(1-i) \sin \left[(1-i) \varphi_{1}\right]-e \cdot i \cos \left(i \varphi_{l}\right) ; \\
\xi_{u}^{\prime}= & \sin \left[(1-i) \varphi_{1}\right] ; \\
\eta_{u}^{\prime}= & \cos \left[(1-i) \varphi_{1}\right] .
\end{aligned}
$$

After development, the enwrapping surface, is reduced to

$$
\begin{equation*}
u=e \frac{i}{i-1} \sin \varphi_{1} . \tag{22}
\end{equation*}
$$

The (15) and (23) equations, in different H planes, represent the conical polyform shaft's profile.

## 2. The Generation of the Exterior Conical Polyform Shaft Using a Cylindrical Enclosing Abrasive Tool

There is the possibility of generating polyform shafts using the interior surface of the abrasive tool, in correlation to the system presented in figure 2 , in which the half-finished material has a planetary motion, related to the $\Delta_{1}$ and $\Delta_{2}$ axes, the last axis being the axis of the generated surface.

The abrasive tool has a rotation motion around its axis, to ensure a parametrical speed according to the technological necessities.


Fig. 2. Reference systems

## - Reference Systems

The following reference systems are defined:

- xyz is a fix system, joined to the crank's rotation axis, $\Delta_{2}$;
- $X_{1} Y_{1} Z_{1}$ - a mobile system, joined to the interior surface of the abrasive tool;
- XYZ - a mobile system, joined to the crank's rotation axis;
- $\xi \eta \zeta$ - a mobile system, joined to the surface of the generated polyform surface;
- $\mathrm{x}_{0} \mathrm{y}_{0} \mathrm{z}_{0}$ - a mobile system, joined to the polyform surface axis, in the planetary motion.


## - The Kinematics of the Generation Process

The kinematics includes the following motions:

- the half-finished material's rotation around its axis, (in the figure the $\zeta$ axis), the $\Delta_{2}$ axis;
- the crank's rotation, here the $\Delta_{1}$ axis, in correlation with the motion around the $\Delta_{1}$ axis;
- the cutting motion, the rotation of the abrasive tool, around $Z_{1}$ axis, being $a$ technological motion.

The generation motions are given by:

$$
\begin{equation*}
x_{0}=\omega_{3}^{T}\left(\varphi_{1}\right) \xi, \tag{23}
\end{equation*}
$$

meaning the shaft's rotation around its axis:

$$
\begin{equation*}
x=\omega_{3}^{T}\left(-\varphi_{2}\right) X \tag{24}
\end{equation*}
$$

being the crank's motion.
There are also defined the corelations between the fix reference systems:

$$
X=x_{0}-b, b=\left\|\begin{array}{l}
e  \tag{25}\\
0
\end{array}\right\|
$$

and

$$
\begin{equation*}
X_{1}=\alpha(x-B) \tag{26}
\end{equation*}
$$

with:
$\alpha=\left\|\begin{array}{ccc}\cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha\end{array}\right\|$ and $B=\left\|\begin{array}{l}\Delta \\ 0 \\ 0\end{array}\right\|$.
In (27) and figure 2, the relationship between the constant values is defined,

$$
\begin{equation*}
\Delta=\frac{R}{\cos \alpha}-\frac{D}{2}-e, \tag{28}
\end{equation*}
$$

where:

- D is the diameter of the polyform shaft;
- e - the length of the planetary mechanism's crank;
- R - the interior radius of the abrasive tool.

From (23), (24), (25) and (26), the motion towards the reference system of the polyform shaft is determined by
$\xi=\omega_{3}\left(\varphi_{1}\right)\left\{\omega_{3}\left(-\varphi_{2}\right)\left[\alpha^{T} X_{1}+B\right]+b\right\}$,
which, after development, will represent the revolution surfaces family by R radius, in the polyform shaft's reference system.

The following form results:

$$
\left\|\begin{array}{c}
\xi \\
\eta \\
\zeta
\end{array}\right\|=\left\|\begin{array}{ccc}
\cos \varphi_{1} & \sin \varphi_{1} & 0 \\
-\sin \varphi_{1} & \cos \varphi_{1} & 0 \\
0 & 0 & l
\end{array}\right\|\left\{\begin{array}{ccc}
\| \cos \varphi_{2} & -\sin \varphi_{2} & 0 \\
\sin \varphi_{2} & \cos \varphi_{2} & 0 \\
0 & 0 & l
\end{array} \|+\right.
$$

$$
\left.\left[\left\|\begin{array}{ccc}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{array}\right\|\left\|\begin{array}{c}
X_{1} \\
Y_{1} \\
Z_{1}
\end{array}\right\|+\left\|+\begin{array}{c}
\Delta \\
0 \\
0
\end{array}\right\|\right]+\left\|\begin{array}{c}
e \\
0 \\
0
\end{array}\right\|\right\}
$$

- The Family of Generating Planes

The parametric equations of the cylindrical surface are defined (the generating body):

$$
C \left\lvert\, \begin{align*}
& X_{I}=-R \cos \theta ;  \tag{31}\\
& Y_{I}=R \sin \theta ; \\
& Z_{I}=t
\end{align*}\right.
$$

with t and $\theta$ independent variable parameters. Thus, after bringing the (31) equations into the (30) transformation, the following form results:
$\left\|\begin{array}{l}\xi \\ \eta \\ \zeta\end{array}\right\|=\left\|\begin{array}{ccc}\cos \varphi_{1} & \sin \varphi_{1} & 0 \\ -\sin \varphi_{1} & \cos \varphi_{1} & 0 \\ 0 & 0 & 1\end{array}\right\| \begin{gathered}A(\theta) \cos \varphi_{2}-B(\theta) \sin \varphi_{2}+e \| \\ A(\theta) \sin \varphi_{2}+B(\theta) \cos \varphi_{2} \\ C(\theta)\end{gathered}$
where the functions $A(\theta), B(\theta)$ and $C(\theta)$ have the following meanings:

$$
\begin{align*}
& A(\theta)=-R \cos \theta \cos \alpha-t \sin \alpha+\Delta \\
& B(\theta)=R \sin \theta  \tag{33}\\
& C(\theta)=-R \cos \theta \sin \alpha+t \cos \alpha
\end{align*}
$$

After development, the surfaces family is determined:

$$
(C)_{\varphi_{1}} \left\lvert\, \begin{align*}
& \xi=A(\theta) \cos \left(\varphi_{2}-\varphi_{l}\right)-B(\theta) \sin \left(\varphi_{2}-\varphi_{l}\right)+e \cos \varphi_{1} ; \\
& \eta=A(\theta) \sin \left(\varphi_{2}-\varphi_{I}\right)+B(\theta) \cos \left(\varphi_{2}-\varphi_{l}\right)-e \sin \varphi_{l} ; \\
& \zeta=C(\theta) . \tag{34}
\end{align*}\right.
$$

The envelope of the surfaces family (34) represents the periphery surface of the polyform shaft.

The cross section of the shaft is determined in the plane:
$\zeta=\mathrm{H}$ ( $\mathrm{H}-$ arbitrary value).
Taking into account the (34) equations, the following condition results:
$H=-R \cos \theta \sin \alpha+t \cos \alpha-\Delta \sin \alpha$
or:

$$
\begin{equation*}
t=\frac{H+\Delta \sin \alpha}{\cos \alpha}+R \operatorname{tg} \alpha \cos \theta . \tag{36}
\end{equation*}
$$

The relationship between the parameters of the rotation angles is also defined

$$
\begin{equation*}
\varphi_{2}=\mathrm{i} \varphi_{1}, \tag{38}
\end{equation*}
$$

with i - the transmission ratio.

## - The enwrapping condition

Thus, the spatial problem can be looked at as if it was a plane one. The enwrapping condition of the plane trajectories method may be used,

$$
\begin{equation*}
\frac{\xi_{Q_{i}}^{\prime}}{\xi_{\theta}^{\prime}}=\frac{\eta_{\varphi_{i}}^{\prime}}{\eta_{\theta}^{\prime}}, \tag{39}
\end{equation*}
$$

for which the partial differentials are defined:
$\xi_{\theta}^{\prime}=A^{\prime}(\theta) \cos (i-1) \varphi_{1}-B^{\prime}(\theta) \sin (i-1) \varphi_{1}-\frac{d t}{d \theta} \sin \alpha ;$
$\eta_{\theta}^{\prime}=A^{\prime}(\theta) \sin (i-1) \varphi_{I}+B^{\prime}(\theta) \cos (i-1) \varphi_{l}-\frac{d t}{d \theta} \sin \alpha ;$
$\xi_{\varphi_{i}}^{\prime}=-(i-1) A(\theta) \sin \left[(i-1) \varphi_{I}\right]-$
$-(i-1) B(\theta) \cos \left[(i-1) \varphi_{1}\right]-e \sin \varphi_{1} ;$
$\eta_{\varphi_{,}}^{\prime}=(i-1) A(\theta) \cos \left[(i-1) \varphi_{1}\right]-$
$-(i-1) B(\theta) \sin \left[(i-1) \varphi_{1}\right]-e \cos \varphi_{1} ;$
$\frac{d t}{d \theta}=-R \operatorname{tg} \alpha \sin \theta$.
The equations, (33), (34) and (39)-(41), represent the conical polyform shaft's profile.

## - The program's algorithm

According to the algorithm previously presented, a soft has been made for the calculus of the profiles' cross sections (see figure 3).


Fig. 3. The program's algorithm

## 3. The Modeling of the Generation of a Polyform Hole Using a Revolution Abrasive Tool

According to figure 4, a generation method of a polyform hole can be imagined, using the interior grinding broach, equipment which is found on the universal grinding machines.

The abrasive tool has two motions:

- the rotation around its axis, the cutting motion (I);
- the feed along the generation line of the generated surface - II, alternated rectilinear motion, which, in reality, is done by the longitudinal table of the grinding machine.


Fig. 4. The generation kinematics and reference systems

The relationship between the constants from the process is obvious:

$$
\begin{equation*}
\Delta=-\frac{R}{\cos \alpha}-e+\frac{D}{2} \tag{42}
\end{equation*}
$$

where:

- $\quad \mathrm{D}$ is the diameter of the polyform hole;
- e - the length of the planetary mechanism's crank;
- R - the radius of the abrasive tool.

According to the reference systems associated to the abrasive tool and the generated surface, the relative motion is defined (see the previous case too)
$\xi=\omega_{3}\left(\varphi_{1}\right)\left\{\omega_{3}\left(-\varphi_{2}\right)\left[\alpha^{T} X_{1}+B\right]+b\right\}$
where:
$\alpha=\left\|\begin{array}{ccc}\cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha\end{array}\right\| ; B=\left\|\begin{array}{c}-\Delta \\ 0 \\ 0\end{array}\right\| ; b=\|-e\|$.
The algorithm of the modeling of the polyform hole surface is similar to the one presented at the previous chapter.

## 4. Numerical Applications

Examples of the application of these algorithms are presented for the modeling of conical polyform surfaces using the two methods previously shown.

In figure 5, it is presented an example of a polyform surface for:
$\mathrm{e}=1[\mathrm{~mm}]$,
$\mathrm{D}=32$ [mm],
$\mathrm{R}=5,5[\mathrm{~mm}]$,
$\alpha=3\left[^{\circ}\right]$, $\mathrm{i}=3 / 4$.


Fig. 5. The polyform hole
In figure 6 and table 1, there are shown the cross section's form and its coordinates.


Fig. 6. The cross section of the polyform hole

Table 1

| $\boldsymbol{\xi}[\mathbf{m m}]$ | $\boldsymbol{\eta}$ [mm] |
| :---: | :---: |
| -13.943 | 0.038 |
| -13.943 | 0.186 |
| -13.942 | 0.301 |
| -13.942 | 0.372 |
| -13.941 | 0.449 |
| -13.938 | 0.746 |
| -13.931 | 1.159 |
| -13.924 | 1.419 |
| -13.919 | 1.603 |
| -13.913 | 1.787 |
| -13.909 | 1.899 |
| -13.869 | 2.710 |
| -13.855 | 2.930 |
| -13.839 | 3.151 |
| -13.816 | 3.439 |


| $\xi[\mathbf{m m}]$ | $\boldsymbol{\eta}[\mathbf{m m}]$ |
| :---: | :---: |
| -13.796 | 3.658 |
| -13.778 | 3.838 |
| -13.759 | 4.020 |
| -13.619 | 5.055 |
| -13.596 | 5.197 |
| -13.564 | 5.379 |
| -13.525 | 5.589 |
| -13.429 | 6.051 |
| -13.254 | 6.752 |
| -13.076 | 7.348 |
| -12.803 | 8.112 |
| -12.461 | 8.901 |
| -12.038 | 9.708 |
| -11.737 | 10.203 |
| -11.411 | 10.684 |

An example related to the algorithm that uses as a generating element a plane surface for the following conditions, see figure 7 and figure 8 (the cross section), along with table 2 :
$\mathrm{A}=14,925[\mathrm{~mm}]$, $\alpha=3\left[{ }^{\circ}\right]$,
$\mathrm{e}=1$ [mm],
$\mathrm{i}=4 / 3$,
$D=27,85[\mathrm{~mm}]$.


Fig. 7. The solid model of $a$ three edged polyform shaft


Fig. 8. The cross section of the polyform shaft

Table 2

| $\boldsymbol{\xi}[\mathbf{m m}]$ | $\boldsymbol{\eta}[\mathbf{m m}]$ |
| :---: | :---: |
| -13.925 | 0.037 |
| -13.920 | 0.495 |
| -13.908 | 0.876 |
| -13.891 | 1.257 |
| -13.867 | 1.635 |
| -13.830 | 2.088 |
| -13.792 | 2.462 |
| -13.689 | 2.833 |
| -13.632 | 3.639 |
| -13.632 | 3.275 |
| -13.558 | 4.070 |
| -13.475 | 4.495 |
| -13.401 | 4.844 |
| -13.305 | 5.255 |
| -13.203 | 5.658 |


| $\boldsymbol{\xi}[\mathbf{m m}]$ | $\boldsymbol{\eta}[\mathbf{m m}]$ |
| :---: | :---: |
| -13.076 | 6.118 |
| -12.941 | 6.565 |
| -12.778 | 7.060 |
| -12.607 | 7.537 |
| -12.359 | 8.163 |
| -12.125 | 8.699 |
| -11.884 | 9.205 |
| -11.589 | 9.769 |
| -10.624 | 11.295 |
| -10.169 | 11.878 |
| -9.320 | 12.779 |
| -9.152 | 12.931 |
| -9.069 | 13.003 |
| -1.266 | 10.330 |
| -0.968 | 10.801 |

A polyform four edged shaft is also presented, similarly modeled:
$\mathrm{A}=15$ [mm],
$\alpha=3\left[{ }^{\circ}\right]$,
$\mathrm{e}=1$ [mm], $i=3 / 4$,
$\mathrm{D}=28[\mathrm{~mm}]$.


Fig. 9. The solid model of a four edged polyform shaft | 7


Fig. 10. The cross section of the polyform shaft
The coordinates of 30 points from the [A,B] interval are shown in table 3 .

The modification of the constant values $\Delta, \alpha$ and D allows the modeling, based on the previous algorithm, of a wide field of surfaces which bear the characteristics of a polyform surface.

Table 3

| $\boldsymbol{\xi}[\mathbf{m m}]$ | $\boldsymbol{\eta}[\mathbf{m m}]$ |
| :--- | :---: |
| -14.000 | 0.027 |
| -13.988 | 0.851 |
| -13.935 | 1.964 |
| -13.804 | 3.408 |
| -13.643 | 4.578 |
| -13.431 | 5.739 |
| -13.243 | 6.575 |
| -13.071 | 7.241 |
| -12.892 | 7.856 |
| -12.831 | 8.049 |
| -12.740 | 8.328 |
| -12.559 | 8.844 |
| -12.499 | 9.003 |
| -12.455 | 9.119 |
| -12.410 | 9.231 |


| $\boldsymbol{\xi}[\mathbf{m m}]$ | $\boldsymbol{\eta}[\mathbf{m m}]$ |
| :---: | :---: |
| -12.295 | 9.514 |
| -12.238 | 9.646 |
| -12.183 | 9.772 |
| -12.089 | 9.978 |
| -12.076 | 10.006 |
| -12.038 | 10.088 |
| -11.975 | 10.217 |
| -11.939 | 10.290 |
| -11.892 | 10.382 |
| -11.751 | 10.646 |
| -11.692 | 10.749 |
| -11.638 | 10.841 |
| -11.310 | 11.317 |
| -11.288 | 11.336 |
| -10.758 | 11.687 |

## 5. The Modeling of a Joint between the Conical Polyform Surfaces

The use of conical polyform surfaces is dictated by the possibility to make a fit between shafts and holes of this type.

Thus, an example, in cross section, between two surfaces of this type is shown in figure 11.


Fig. 11. A joint's errors
The fact that the two surfaces are not identical is obvious.

The dimensional error, measured in plane coordinates, is shown in table 4 where:
$\rho_{\mathrm{a}}$ is the dimension of the shaft in mm ;
$\rho_{\mathrm{A}}$ - the dimension of the hole in mm ;
$\theta$ - the center angle.
Table 4

| $\boldsymbol{\rho}_{\mathbf{A}}[\mathbf{m m}]$ | $\boldsymbol{\rho}_{\mathbf{a}}[\mathbf{m m}]$ | $\boldsymbol{\theta}{ }^{\mathbf{0}} \mathbf{}$ ] | $\boldsymbol{\rho}_{\mathbf{A}}-\mathbf{\rho}_{\mathbf{a}}[\mathbf{m m}]$ |
| :---: | :---: | :---: | :---: |
| 18,449 | 18,850 | 0 | $-0,400$ |
| 17,986 | 18,023 | 20 | $-0,036$ |
| 16,891 | 17,022 | 40 | $-0,130$ |
| 16,250 | 16,650 | 60 | $-0,400$ |
| 16,891 | 17,022 | 80 | $-0,130$ |
| 17,986 | 18,023 | 100 | $-0,036$ |

## Conclusions

The modeling of conical polyform surfaces, based on the shown kinematics, proves the viabilty, geometricaly at least, of the presented solutions.

Obviously, not any length of polyform surface can be made by this kind of generation methods.

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## Modelarea generării suprafețelor poliforme conice

## Rezumat

Lucrarea prezintă modalități diferite pentru generarea suprafețelor poliforme conice prin rectificare, utilizând suprafețele active ale unor corpuri abrazive: plan, cilindric interior şi cilindric exterior, într-o cinematică planetară. Modelarea generării $s-a$ făcut în baza teoriei suprafețelor în înfăşurare - metoda traiectoriilor plane de generare.

Se propune o soluție pentru generarea suprafețelor cuprinsă şi cuprinzătoare, în vederea formării unui ajustaj.

Se prezintă, de asemenea, exemple numerice.

## La modèlisation de la génération des surfaces polyformes coniques

## Résumé

On presente dans cet ouvrage différentes méthodes pour modèler un joint entre deux surfaces polyformes coniques.

Les méthodes peuvent être facilement appliquées sur les machines outiles universelles et ont l'avantage d'être des solutions simples qui mènent à une précision élevée dans la génération des surfaces. Aussi, ont presente des applications numériques.

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