Active Control of Convection in a Melted Material Layer
I. Convection in an Infinite Fluid Layer

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ABSTRACT

This paper is the first part of a study of the active control of Bénard-Marangoni convection of an infinite fluid layer heated from bellow with a constant heat flux. A linear proportional control method is used to perturb the lower boundary heat flux proportional to the local amplitude of a shadowgraph measurement. Linear stability analyses was used to establish that the onset of Bénard-Marangoni convection can be delayed using a linear proportional control method; furthermore, the active control parameters necessary for an established result can be obtained.
Keywords: active control, melted material layer, convection.

1. Introduction

Since Pearson [1] proved the phenomenon of surface tension-driven convection and Nield [2] the co-existence of buoyancy and surface tension-driven convection, many researchers studied these phenomena trying to establish clear connection between them. The Rayleigh-Bénard convection and Bénard-Marangoni convection do reinforce each other in the context of the assumption that the upper fluid boundary remains flat [2]. Even if this simplification was already questioned in several studies and Scriven and Sternling [3] concluded that the surface deformability may render the layer of fluid unstable under virtually all conditions, the flat upper boundary condition is an assumption which, used in the theoretical studies of Bénard-Marangoni convection, meets good verification [4-5]. This paper is also considering that the upper surface is flat. For small values of Rayleigh numbers (3.0e-7) and thin fluid layers, Bénard-Marangoni convection is responsible for the pattern formation. If the experimental work questioned the stability of patterns which arise and the existence of an unique value for Marangoni number, numerical modeling tried and confirmed the experimental results: the energy stability theory [6], the linear stability analysis [1], [2] and bifurcation analysis [7-8]. This study is a linear stability analysis of a linear proportional method for the active control of Bénard-Marangoni convection.

Rayleigh-Bénard convection and Bénard-Marangoni convection are not always desired phenomena in industrial applications. The delay or suppress of convection in Rayleigh-Bénard and Bénard-Marangoni convection has received a great attention in the last period [9÷10].

Theoretical studies [11÷14] as well as experimental works [15÷16] analyzed different methods of control of Rayleigh-Bénard convection. The successful results of Tang and Bau [17] initiated the active control of Rayleigh-Bénard convection through the control of temperature or heat flux at the fluid lower boundary. According to the classification established by Gad-el-Hak [18], these are active reactive feedback control methods.

This work is studying the active control of surface tension-induced convection of infinite horizontal fluid layers heated from bellow with a constant flux, situation which corresponds experimentally to a novel shadowgraphic system.

2. Mathematical formulation

In a Boussinesq infinite fluid layer, the linear analysis leads to the stability equation [13], [19] for the perturbation temperature amplitude, Θ:

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \sigma\right)\left(D^2 - a^2 - \frac{\sigma}{Pr}\right)\Theta = = -a^2 R\Theta, \qquad (1)$$

where a is the wave number, σ is the growth rate, R is the Rayleigh number, Pr is the Prandtl number and D is the notation for $D = \partial / \partial a$.

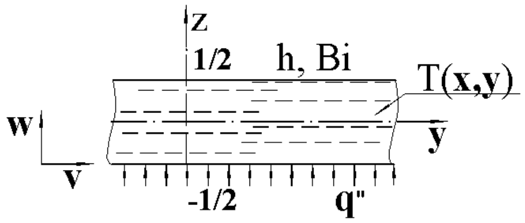

Fig. 1. Infinite fluid layer.

The solution of equation (1) is:

$$\Theta = \sum_{i=1}^{3} \left(E_i \cosh(x_i z) + O_i \sinh(x_i z) \right) \qquad (2)$$

where x_i i=1…3, the roots of the characteristic equation

$$\left(x_i^2 - a^2\right)\left(x_i^2 - a^2 - \sigma\right)\left(x_i^2 - a^2 - \frac{\sigma}{Pr}\right)$$

$$+ a^2 R = 0 . \qquad (3)$$

The six boundary conditions necessary for finding the coefficients E_i and O_i and, consequently, the perturbation temperature amplitude, Θ, are:

- the no penetration condition, applied at upper and lower boundary,

$$\left(D^2 - a^2 - \sigma\right)\Theta\left(\pm \frac{1}{2}\right) = 0 ; \qquad (4)$$

- the no slip condition, applied at the lower boundary,

$$D\left(D^2 - a^2 - \sigma\right)\Theta\left(-\frac{1}{2}\right) = 0 ; \qquad (6)$$

- the thermal transfer condition, applied at the upper boundary,

$$D\Theta\left(\frac{1}{2}\right) + Bi\Theta\left(\frac{1}{2}\right) = 0 ; \qquad (7)$$

- the condition of surface-tension driven convection,

$$D^2\left(D^2 - a^2 - \sigma\right)\Theta\left(\frac{1}{2}\right) = a^2 Ma\Theta\left(\frac{1}{2}\right); \qquad (8)$$

applied at the upper boundary. Bi is the Biot number, the Marangoni number $Ma = \frac{d\gamma_s \overline{q}}{k_f K_f \rho \nu}$, d is the fluid layer thickness, $\overline{q}$ is the medium heat flux, k_f is the fluid thermal diffusivity, K_f is the fluid thermal conductivity, ρ is the fluid density, ν is the fluid viscosity and γ_s is thermal coefficient of surface tension.

- the sixth thermal boundary condition is an expression of the control method applied to the system, the heat flux proportional to the local shadowgraphic signal:

$$q = \overline{q}\left(1 + g_p \frac{\delta\rho}{\rho_0}\right), \qquad (9)$$

where g_p is the proportional gain, , $\delta\rho / \rho_0$ is the relative variation of the fluid density, ρ_0 is the fluid density at temperature T_0.

The dimensionless form of equation (9), in terms of perturbation temperature amplitude is [14]:

$$D\Theta\left(-\frac{1}{2}\right) = \gamma a^2 \int_{-1/2}^{1/2} \Theta dz , \qquad (10)$$

where $\gamma = -2g_p \frac{H}{d}\frac{\partial \eta_0}{\partial \Theta}$, d is the fluid layer height, H is the distance between the shadowgraph and convection layer, η is the ratio of the thermal diffusivities of the lower boundary layer and the fluid, K_l/K_f.

The boundary conditions (4)÷(8) and (10) applied to equation (2), gives the condition for the onset of convection:

$$D = \begin{vmatrix} A_1 & A_2 \\ A_3 & A_4 \end{vmatrix} = 0 , \qquad (11)$$

where

$$A_1 = \begin{bmatrix} X_1C_1 & X_2C_2 & X_3C_3 \\ X_1C_1 & X_2C_2 & X_3S_3 \\ -x_1X_1S_1 & -x_2X_2S_2 & -x_3X_3S_3 \end{bmatrix}, \qquad (12)$$

$$A_2 = \begin{bmatrix} X_1S_1 & X_2S_2 & X_3S_3 \\ -X_1S_1 & -X_2C_2 & -X_3S_3 \\ x_1X_1C_1 & x_2X_2C_2 & x_3X_3C_3 \end{bmatrix}, \qquad (13)$$

$$A_3 = \begin{bmatrix} x_{1m}C_1 & x_{2m}C_2 & x_{3m}C_3 \\ x_1S_1 + BiC_1 & x_2S_2 + BiC_2 & x_3S_3 + BiC_3 \\ \frac{\left(2\gamma a^2 + x_1^2\right)S_1}{x_1} & \frac{\left(2\gamma a^2 + x_2^2\right)S_2}{x_2} & \frac{\left(2\gamma a^2 + x_2^2\right)S_2}{x_2} \end{bmatrix} \qquad (14)$$

$$A_4 = \begin{bmatrix} x_{1m}S_1 & x_{2m}S_2 & x_{3m}S_3 \\ x_1C_1 + BiC_1 & x_2C_2 + BiC_2 & x_3C_3 + BiS_3 \\ -x_1C_1 & -x_2C_2 & -x_3C_3 \end{bmatrix} \qquad (15)$$

where

$$X_i = x_i^2 - a^2 - \sigma, i{=}1\ldots3;$$

$$x_{im} = x_i^2 X_i - a^2 Ma , i{=}1\ldots3.$$

Considering the real part $\Re(\sigma) = 0.0$ as marginal state, the whole branch of solution is found using continuation method [20].

3. Simulation results

In all the numerical applications of this paper, we used the following values for Rayleigh, Prandtl and Biot numbers: Ra = 3.0e-7, Pr = 100 and Bi = 0.001. For the ideal case of the active control of convection of an infinite fluid layer without lower boundary, the loss of stability through complex values of the growth rate couldn't be find.

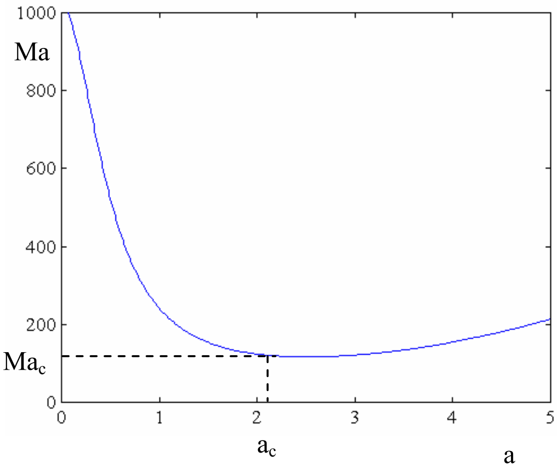

Fig. 2. Ma — a dependence for γ=20.

Figure 2 shows the variation of Marangoni number as a function of the wave number for a proportional gain γ=20. The state corresponding to the minimum Marangoni number is the critical state.

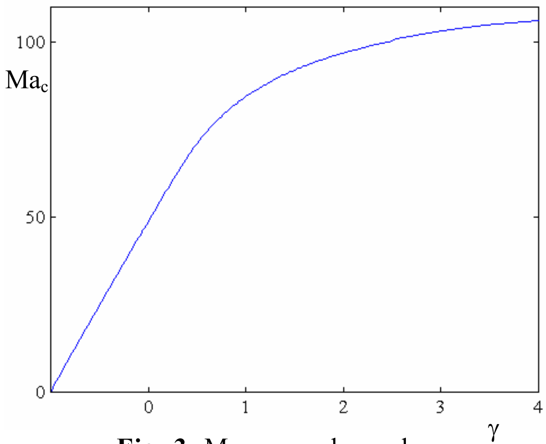

Fig. 3. Ma_c — γ dependence.

Figures 3÷4 show the variation of critical Marangoni number and critical wave number as a function of the proportional gain. We are concluding that the onset of convection can be moved toward higher values of Marangoni number increasing the proportional gain.

Figure 4 suggests an increase of the wavenumber of the patterns that appear with the proportional gain and shows a positive gain at which the minimum wavenumber occurs.

These results show that the critical state for the onset of Bénard-Marangoni convection of an infinite fluid layer can be controlled according to the controller proportional gain.

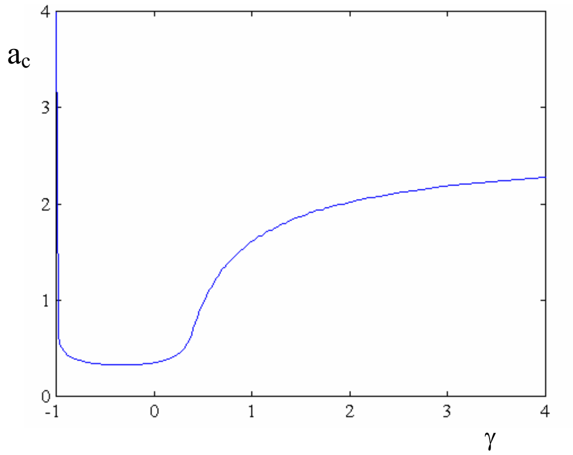

Fig. 4. Critical wave number a_c as a function of the controller gain γ.

Linear stability analysis was used to reproduce the experimental conditions found at Pearson [1], Koschmider and Switzer [21], Palmer and Berg [22] and these results are in good agreement with their experimental values. Taking a small Rayleigh number, the surface tension-driven convection curves obtained by Pearson [1] were compared with these results.

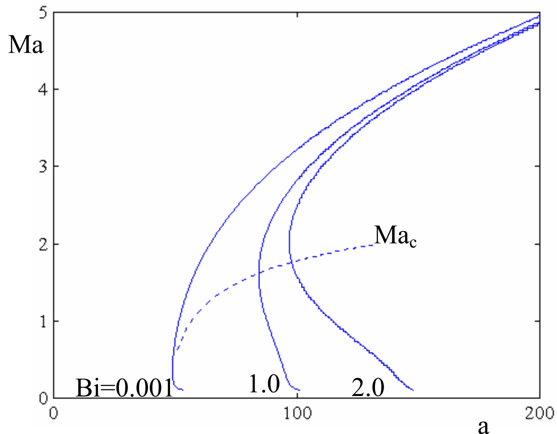

Fig. 5. Ma — a dependence for Biot numbers Bi=0.001, 1 and 2. The variation of the minimum of these curves as a function of Biot number is also shown in the figure.

The Marangoni number — wave number dependence for Biot number 0.001, 1 and 2 as well as the evolution of the critical states (corresponding to the minimum Marangoni number), Fig. 5, shows a very good agreement.

4. Conclusions

Using the linear stability analysis, this paper studies the onset of convection of an infinite fluid layers heated from below with a constant heat flux. Considering microgravity conditions for the numerical simulation, Bénard-Marangoni convection is the driven phenomenon of the pattern formation.

For this case, the critical state can be shifted towards higher values of Marangoni number, which represents the task of active control and its applications. It was noticed that the system can loose stability only through real eigenvalues towards steady-state convection.

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Controlul Activ al Convecției într-un Strat de Material Topit
I. Convecția într-un Strat Infinit

REZUMAT

Acestă lucrare este prima parte a unui studiu asupra convecției Bénard-Marangoni într-un strat de fluid încălzit de un flux constant. O metodă de control linear proporțională este folosită pentru a perturba fluxul de căldură proporțional cu măsurătorile unui sistem de măsurare. S-a observat că inițierea convecției Bénard-Marangoni poate fi întârziată folosind o metodă de control proporțională şi că parametrii de control necesari pentru obținerea unui anumit rezultat pot fi stabiliți.

Aktive Steuerung der Konvektion in einem geschmolzenen materiellen Layer. I. Konvektion in einer endlosen flüssigen Schicht

AUSZUG

Dieses Papier ist das erste Teil einer Studie der aktiven Steuerung von Bénard-Marangoni Konvektion einer endlosen flüssigen Schicht, die vom Gebrüll mit einem konstanten Hitzefluß geheizt wird. Eine lineare proportionale Steuermethode wird verwendet, um den niedrigeren Grenzhitzefluß zu stören, der zum lokalen Umfang eines Shadowgraphmaßes proportional ist. Lineare Stabilität Analysen wurden verwendet, um herzustellen, daß der Angriff der Bénard-Marangoni Konvektion mit einer linearen proportionalen Steuermethode verzögert werden kann; ausserdem können die aktiven Steuerparameter, die für ein hergestelltes Resultat notwendig sind, erhalten werden.